Probabilistic context-free grammars, garden-pathing, and surprisal

Roger Levy 9.19: Computational Psycholinguistics

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Arabic short vowels and consonant lengths



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bopomofo phonetic symbols (used in Taiwan for Mandarin)

Gwoin Tzyhmuu/ 夕Віў 夕Рід Пи́ Х СБ'й Л VIII **З** N би 力Lib **ろK本 乙NG圖電厂H林** 4」「基人CHI版 广GN团集 TSEE# PSHH DRH HI to A CHG 5TS# 思[以上分升] **この影さた**4 ERIA ENGRI (出行户目卫与上明石二大时,加)+1 强母。) [×上村丹] 为处门等关系一人,名法通疗性,七年十一月二十三日丝菌群公布(体名注意 了些,十九年四月世九月四天北司令法今名)。 BPM等及第二人,名田北市為下,十七年九月廿六日大甲代公布。 重的周宇,关注母的法法;禁止于言法;甚至在如言的字片,主,并后曰:(调音 不用无相当的平母,作•马之山 姜-天的春波 推音比鞋 隔音,十;上背,√;法背,丶;入背,●(圖皆不用)。 第二点的整洁 末用石板 推定计法法未示 另有段明。

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Word boundary markers



bopomofo phonetic symbols (used in Taiwan for Mandarin)



豢→天约を注 だう此間,陽子,十;上背,√;法背,丶;八背、•(圖皆不用)。

3 (Arabic figure due to BoogaLoue bopomofo due to <u>厂 X 左 力 |</u>; both licensed under CC BY-SA)

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 - Brown Corpus, 1989 Wall Street Journal, spoken Switchboard
- There are now treebanks in dozens of languages!

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The 1987 statute Mrs. Yeargin violated was designed to enforce provisions of South Carolina's school-improvement laws



long-distance dependencies

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• Annotations are often "flatter" than often (theoretically) ideal



Penn Treebank phrasal categories

- 1 ADJP Adjective phrase
- 2 ADVP Adverb phrase
- 3 NP Noun phrase
- 4 PP Prepositional phrase
- 5 S Simple declarative clause
- 6 SBAR Clause introduced by subordinating
- 7 SBARQ Direct question introduced by wh-word or
- 8 SINV Declarative sentence with subject-auxiliary
- 9 SQ Subconstituent of SBARQ excluding wh-word
- 10 VP Verb phrase
- 11 WHADVP Wh-adverb phrase
- 12 WHNP Wh-noun phrase
- 13 WHPP Wh-prepositional phrase
- 14 X Constituent of unknown or uncertain

There are some other phrasal categories to annotate spoken transcripts, in the Switchboard part of the Penn Treebank, too

Penn Treebank tagset

 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 	JJ Adjective JJR Adjective, comparative JJS Adjective, superlative LS List item marker MD Modal NN Noun, singular or mass NNS Noun, plural NNP Proper noun, singular NNPS Proper noun, plural PDT Predeterminer POS Possessive ending PRP Personal pronoun PP Possessive pronoun RB Adverb RBR Adverb, comparative RBS Adverb, superlative RP Particle	31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47.	VBP VBZ WDT WP WRB # \$,: ()	Verb, non-3rd ps. sing. present Verb, 3rd ps. sing. present wh-determiner wh-pronoun Possessive wh-pronoun wh-adverb Pound sign Dollar sign Sentence-final punctuation Comma Colon, semi-colon Left bracket character Right bracket character Straight double quote Left open single quote Left open double quote Right close single quote
23.	RP Particle	47.	т	Right close single quote
24.	SYM Symbol (mathematical or scientific)	48.	П	Right close double quote

• Spaces delimit word boundaries



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Incidentally: these are S interjection & disfluency markers, \boldsymbol{k} used in annotating speech -DFL-NP-SBJ VP INTJ П н VP E_S PRP MD UH I. 'd VB NP Like she CC NP NP eat NN ΝN and NNS Т cookies ice cream

Spaces delimit word boundaries



 All tree leaves (words and empty categories) are dominated by their part-of-speech tag alone

Spaces delimit word boundaries



- All tree leaves (words and empty categories) are dominated by their part-of-speech tag alone
- You can treat Treebank annotations (mostly) as derivations trees from a context-free grammar, BUT best to treat the annotations as *information about syntactic syntactic structure* that we want grammars that will accurately recover

Software for searching treebanks: Tregex

Tree-matching pattern



Syntactic ambiguity

https://languagelog.ldc.upenn.edu/nll/?p=17431

Syntactic ambiguity

 Context-free grammars predict multiple derivations for many word strings
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 But CFGs don't explain where our interpretation preferences come from

A friend of Mary's husband wanted to visit and look over our garden last Tuesday.

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Question

Syntax

Proportion of choices

Who wanted to visit and see our garden?



Someone else

A friend of Mary's husband wanted to visit and look over our garden last Tuesday.

Question **Syntax Proportion of choices** Who wanted to visit and see our garden? NP NN NP Det NN PP husband The husband of one of 0% Mary's friends friend А IN NP NNP POS of Mary 's NP NΡ PP Someone who is friends with 100% NN IN NP Det Mary's husband I. А friend of NP NN NNP POS husband н 's Mary

A friend of Mary's husband wanted to visit and look over our garden last Tuesday.



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Question Syntax People choosing

How does "Last Tuesday" relate to the rest of the sentence?

This was the time that the person's desire (to visit and learn about our garden) arose

This was the person's preferred time both to visit and to look over our garden

This was the person's preferred time to look over our garden



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Something else

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- Yet 66% of respondents chose this analysis:



- 18% preferred an analysis differing in only 1 ambiguity
- 18% preferred analysis differing in 2 ambiguities
- Theoretical challenge: what determines the "preferred" analysis, and how do we find it?

*recommended question: why 20, not 2×3×2×2=24?



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• Syntactic:

Jamie was clearly intimidated...



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Jamie was clearly intimidated ... by [source]



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Jamie was clearly intimidated... by [source]

Phonological knowledge:

Terry ate an...



Previous Input Current Input

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Jamie was clearly intimidated ... by [source]

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Terry ate an... apple/orange/ice cream cone



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The children went outside to...



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The children went outside to...play The squirrel stored some nuts in the...



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The children went outside to...play The squirrel stored some nuts in the...statue



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- 1. Specify precisely the goals of the cognitive system
- 2. Formalize model of the environment to which the cognitive system is adapted
- 3. Make minimal assumptions re: computational limitations
- 4. Derive predicted optimal behavior given 1–3
- 5. Compare predictions with empirical data
- 6. If necessary, iterate 1–5

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Use controlled, experimental case studies to investigate real-time human language understanding

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Putting probabilities on structures

• Some syntactic structures are rarer than others



- We want a model that will probabilistically score parts of a tree
- One simple model for this is the PROBABILISTIC (or STOCHASTIC) CONTEXT-FREE GRAMMAR (PCFG or SCFG)

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Probabilistic Context-Free Grammars

A *probabilistic* context-free grammar (PCFG) consists of a tuple (N, V, S, R, P) such that:

- N is a finite set of non-terminal symbols;
- V is a finite set of terminal symbols;
- ► *S* is the start symbol;
- ▶ *R* is a finite set of rules of the form $X \to \alpha$ where $X \in N$ and α is a sequence of symbols drawn from $N \cup V$;
- ▶ *P* is a mapping from *R* into probabilities, such that for each $X \in N$,

$$\sum_{X \to \alpha] \in R} P(X \to \alpha) = 1$$

PCFG derivations and derivation trees are just like for CFGs. The probability P(T) of a derivation tree is simply the product of the probabilities of each rule application.

Example PCFG

 $\begin{array}{cccc} 1 & S & \rightarrow \mathsf{NP} & \mathsf{VP} \\ 0.8 & \mathsf{NP} & \rightarrow \mathsf{Det} & \mathsf{N} \\ 0.2 & \mathsf{NP} & \rightarrow \mathsf{NP} & \mathsf{PP} \\ 1 & \mathsf{PP} & \rightarrow \mathsf{P} & \mathsf{NP} \\ 1 & \mathsf{VP} & \rightarrow \mathsf{V} \end{array}$

- $1 \quad \text{Det} \to \text{the}$
- $0.5 \text{ N} \rightarrow \text{dog}$
- 0.5 N \rightarrow cat
- $1 \quad P \rightarrow near$
- $1 \quad V \rightarrow \text{growled}$

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 $P(\mathsf{T}) = 1 \times 0.2 \times 0.8 \times 1 \times 0.5 \times 1 \times 1 \times 0.8 \times 1 \times 0.5 \times 1 \times 1$ = 0.032

PCFG review (2)

We just learned how to calculate the probability of a tree

- The probability of a string w_{1...n} is the sum of the probabilities of all trees whose yield is w_{1...n}
- The probability of a string prefix w_{1...i} is the sum of the probabilities of all trees whose yield **begins with** w_{1...i}
- If we had the probabilities of two string prefixes w_{1...i-1} and w_{1...i}, we could calculate the conditional probability P(w_i|w_{1...i-1}) as their ratio:

$$P(w_i|w_{1...i-1}) = \frac{P(w_{1...i})}{P(w_{1...i-1})}$$

Consider the following noun-phrase grammar:

		–	De	
$\frac{2}{3}$	$NP \to Det~N$	$\frac{2}{3}$	Ν	ightarrow dog
$\frac{1}{3}$	$NP o NP \ PP$	$\frac{1}{3}$	Ν	ightarrow cat
ĭ	$PP \to P NP$	${ ilde 1}$	Ρ	ightarrow near

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 $D_{ot} \rightarrow th_{o}$

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		T	De	$\iota \rightarrow \iota \iota \iota \iota$
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$\check{1}$	$PP \to P NP$	ľ	Ρ	ightarrow near

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the...

what is the probability that the next word is *dog*?

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$$P(\log|\text{the}) = \frac{2}{3}$$

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what is the probability that the next word is *dog*? Intuitively, the answers to this question should be

$$P(\log|\text{the}) = \frac{2}{3}$$

because the second word HAS to be either *dog* or *cat*.



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We "should" just enumerate the trees that cover the dog ...,

		1	De	t ightarrow the
$\frac{2}{3}$	$NP \to Det~N$	$\frac{2}{3}$	Ν	ightarrow dog
$\frac{1}{3}$	$NP o NP ext{ PP}$	$\frac{1}{3}$	Ν	ightarrow cat
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... but there are infinitely many trees.



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$$\begin{array}{cccc} 1 & \text{Det} \to \text{the} \\ \frac{2}{3} & \text{NP} \to \text{Det N} & & \frac{2}{3} & \text{N} \to \text{dog} \\ \frac{1}{3} & \text{NP} \to \text{NP PP} & & \frac{1}{3} & \text{N} \to \text{cat} \\ 1 & \text{PP} \to \text{P NP} & & 1 & \text{P} \to \text{near} \end{array}$$

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You can think of a *partial* tree as marginalizing over all completions of the partial tree.

It has a corresponding marginal probability in the PCFG.

NP	NP NP	
Det N	NP PP	NP PP
the dog	Det N P NP	Det N P NP
	the dog near Det N	the dog near Det N
	the dog	the cat
$\frac{4}{9}$	$\frac{8}{81}$	$\frac{4}{81}$

$$\begin{array}{cccc} 1 & \text{Det} \to \text{the} \\ \frac{2}{3} & \text{NP} \to \text{Det N} & & \frac{2}{3} & \text{N} \to \text{dog} \\ \frac{1}{3} & \text{NP} \to \text{NP PP} & & \frac{1}{3} & \text{N} \to \text{cat} \\ 1 & \text{PP} \to \text{P NP} & & 1 & \text{P} \to \text{near} \end{array}$$

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NP	Ν	Р		Γ	VР		
Det N	NP	PP	N	Р]	PP	
the dog	Det N the dog	P NP near Det N	Det the	N dog	P near	NF Det	> N
4 9 NF)	the dog $\frac{8}{81}$				$\frac{4}{81}$	cat
NP Det N <i>the dog n</i>	$\begin{array}{c c} PP \\ \hline P & NP \\ \hline ear & Det \\ \hline the \\ \frac{12}{81} \end{array}$						

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Ν	P	N	1P
NP	PP	NP	PP
Det N	P NP	Det N	P NP
the dog	near Det N	the dog	near Det N
	the dog		the cat
	$\frac{8}{81}$		$\frac{4}{81}$
	NP		
PP	NP PF	C	
NP	Det N		
ar $Det N$	the dog		
$\frac{the}{rac{12}{81}}$	<u>4</u> 27		
	PP PP NP PP NP r $Det N$ r he $\frac{12}{81}$	NP NP PP $Det N P NP$ $the dog near Det N$ $the dog$ $\frac{8}{81}$ NP PP NP $Det N$ $fr Det N$ $the dog$ $\frac{12}{81}$ $\frac{4}{27}$	$NP PP NP PP NP Det N the dog near Det N the dog \frac{8}{81} NP Det N the dog \frac{8}{81} NP PP Det N the dog the \frac{12}{81} \frac{4}{27}$





BUT! These tree probabilities form a geometric series:

$$P(the \ dog \dots) = \frac{4}{9} + \frac{4}{27} + \frac{4}{81} + \frac{4}{243} + \cdots$$



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$$= \frac{2}{3}$$

590

... which matches the original rule probability

$$\frac{2}{3} N \rightarrow dog$$

Generalizing the geometric series induced by rule recursion

In general, these infinite tree sets arise due to *left recursion* in a probabilistic grammar

$$A \to B \alpha \qquad \qquad B \to A \beta$$



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and solve for its closure $R_L = (I - P_L)^{-1}$.

(Stolcke, 1995)

◆□▶ ◆□▶ ◆ ≧▶ ◆ ≧▶ → ≧ → りへぐ

L	ROOT	$ \rightarrow NP$	1	Det	t o the
2	NP	\rightarrow Det N	$\frac{2}{3}$	Ν	ightarrow dog
Í 3	NP	\rightarrow NP PP	$\frac{1}{3}$	Ν	ightarrow cat
	PP	\rightarrow P NP	ı 1	Ρ	ightarrow near

The closure of our left-corner matrix is

		ROOT	NP	PP	Det	Ν	Р
	ROOT	/ 1	$\frac{3}{2}$	0	1	0	0 \
	NP	0	$\frac{3}{2}$	0	1	0	0
D	PP	0	Ō	1	0	0	1
$\kappa_L =$	Det	0	0	0	1	0	0
	Ν	0	0	0	0	1	0
	Р	0	0	0	0	0	1 /

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L	ROOT	$ \rightarrow NP$	1	Det	t ightarrow the
2	NP	\rightarrow Det N	$\frac{2}{3}$	Ν	ightarrow dog
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	PP	0	Ō	1	0	0	1
	Det	0	0	0	1	0	0
	Ν	0	0	0	0	1	0
	Р	0	0	0	0	0	1 /

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▶ Refer to an entry (X, Y) in this matrix as $R(X \stackrel{*}{\Rightarrow}_L Y)$

	ROOT	$\Gamma ightarrow NP$	1	Det	t ightarrow the
2	NP	ightarrow Det N	$\frac{2}{3}$	Ν	ightarrow dog
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-	R00 ⁻	$\Gamma ightarrow NP$	1	De	t ightarrow the
2	NP	\rightarrow Det N	$\frac{2}{3}$	Ν	ightarrow dog
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-	PP	\rightarrow P NP	1	Ρ	ightarrow near

The closure of our left-corner matrix is

		ROOT	NP	PP	Det	Ν	Р
$R_L =$	ROOT	/ 1	$\frac{3}{2}$	0	1	0	0 \
	NP	0	$\frac{3}{2}$	0	1	0	0
	PP	0	Ō	1	0	0	1
	Det	0	0	0	1	0	0
	Ν	0	0	0	0	1	0
	Р	0	0	0	0	0	1/

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- We need to do the same with unary chains, constructing a unary-closure matrix R_U.

We can use the Earley algorithm (Earley, 1970) in a probabilistic incarnation (Stolcke, 1995) to deal with these infinite tree sets.

The (slightly oversimplified) probabilistic Earley algorithm has two fundamental types of operations:

- Prediction: if Y is a possible goal, and Y can lead to Z through a left corner, choose a rule Z → α and set up α as a new sequence of possible goals.
- Completion: if Y is a possible goal, Y can lead to Z through unary rewrites, and we encounter a completed Z, absorb it and move on to the next sub-goal in the sequence.

- Parsing consists of constructing a *chart* of *states* (items)
- A state has the following structure:



- The forward probability is the total probability of getting from the root at the start of the sentence through to this state
- The inside probability is the "bottom-up" probability of the state

Inference rules for probabilistic Earley:

Prediction: $\begin{array}{ccc}
X \rightarrow \beta \circ Y\gamma \\
p & q
\end{array}$ $\begin{array}{ccc}
a : R(Y \stackrel{*}{\Rightarrow}_{L} Z) & b : Z \rightarrow \alpha \\
\hline
Z \rightarrow \circ \alpha \\
abp & b
\end{array}$

Inference rules for probabilistic Earley:

Prediction: $X \rightarrow \beta \circ Y \gamma$ $a: R(Y \stackrel{*}{\Rightarrow}_{L} Z) \quad b: Z \to \alpha$ р q $Z \rightarrow \circ \alpha$ abp b **Completion**: $X \rightarrow \beta \circ Y \gamma$ $Z \rightarrow \alpha \circ$ $a: R(Y \stackrel{*}{\Rightarrow}_{U} Z)$ h С q р $X \rightarrow \beta Y \circ \gamma$ acp acq

Efficient incremental parsing: probabilistic Earley

the



Efficient incremental parsing: probabilistic Earley


```
Det→othe
1 1
NP \rightarrow \circ Det N
\frac{2}{3} \times \frac{3}{2} \frac{2}{3}
NP \rightarrow \circ NP PP
\frac{1}{3} \times \frac{3}{2} \frac{1}{3}
\mathsf{ROOT}{\rightarrow}{\circ}\mathsf{NP}
1 1
                  the
```



dog

near

I → I → the → I → I → QQC

```
Det→othe
1 1
NP \rightarrow \circ Det N
\frac{2}{3} \times \frac{3}{2} \frac{2}{3}
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1
    1
                 the
```

```
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1 1
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\frac{1}{3} \times \frac{3}{2} \frac{1}{3}
\mathsf{ROOT}{\rightarrow}{\circ}\mathsf{NP}
1 1
                  NP \rightarrow Det \circ N
                                    \frac{2}{3}
                  1
                  Det→the∘
                       1
                   1
                the
```

dog

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```
Det→othe
1 1
NP \rightarrow \circ Det N
\frac{2}{3} \times \frac{3}{2} \frac{2}{3}
NP \rightarrow \circ NP PP
\frac{1}{3} \times \frac{3}{2} \frac{1}{3}
\mathsf{ROOT}{\rightarrow}{\circ}\mathsf{NP}
1
    1
                    NP \rightarrow Det \circ N
                                       \frac{2}{3}
                    1
                    Det→the∘
                                      1
                    1
                 the
```

near





near





near



near



near

·□··· the·· · ≥ · · ·

















Prefix probabilities from probabilistic Earley

► If you have just processed word w_i , then the prefix probability of $w_{1...i}$ can be obtained by summing all forward probabilities of items that have the form $X \to \alpha w_i \circ \beta$

▲□▶ ▲圖▶ ▲≣▶ ▲≣▶ ■ ののの

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In our example, we see:

$$P(\text{the}) = 1$$

$$P(\text{the dog}) = \frac{2}{3}$$

$$P(\text{the dog near}) = \frac{2}{9}$$

$$P(\text{the dog near the}) = \frac{2}{9}$$

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Taking the ratios of these prefix probabilities can give us conditional word probabilities

Probabilistic Earley as an "eager" algorithm

- From the *inside probabilities* of the states on the chart, the posterior distribution on (incremental) trees can be directly calculated
- This posterior distribution is *precisely* the correct result of the application of Bayes' rule:

$$P(T_{\text{incremental}}|w_{1...i}) = \frac{P(w_{1...i}, T_{\text{incremental}})}{P(w_{1...i})}$$

- Hence, probabilistic Earley is also performing rational disambiguation
- Hale (2001) called this the "eager" property of an incremental parsing algorithm.

Probabilistic Earley algorithm: key ideas

- We want to use probabilistic grammars for both disambiguation and calculating probability distributions over upcoming events
- Infinitely many trees can be constructed in polynomial time () and space ()
- The prefix probability of the string is calculated in the process
- By taking the log-ratio of two prefix probabilities, the surprisal of a word in its context can be calculated

Probabilistic Earley algorithm: key ideas

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Probabilistic Earley algorithm: key ideas

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• Let's consider another case of ambiguity:

• Let's consider another case of ambiguity:

The complex houses married students and their families.

• Let's consider another case of ambiguity:

The complex houses married students and their families.

The prime number few.

• Let's consider another case of ambiguity:

The complex houses married students and their families.

The prime number few.

 In-class exercise: develop a PCFG in which which the "garden-path" analysis is strongly disfavored

				1	De	t o th	e	
		$\frac{2}{3}$ NP	$P ightarrow Det \ N$	$\frac{2}{3}$	N	ightarrow dc	og	
		$\frac{1}{3}$ NP	$P ightarrow NP \ PP$	$\frac{1}{3}$	N	ightarrow ca	t	
		1 PP	\rightarrow P NP	1	Р	ightarrow ne	ear	
Incrementality: yo	u can thin	k of a <i>part</i>	ial tree as mar	ginaliz	zing	over a	all	
completions of	the nartia	l tree			Ū			
It has a corres	ponding m	arginal pro	bability in the	PCFG).			
NP	Ν	P			Ν	VP		
$\overline{\mathbf{D}_{ot}}$ N	ND				D			
Det N	NP	P P		INI				
the dog	Det N	P NF)	Det	Ν	Р	Ν	Р
			_		•	•	_	
	the dog	<i>near</i> Det	Ν	the	dog	near	Det	Ν
			;				.;	• ,
Δ		the	dog				the	cat
$\frac{4}{9}$		$\frac{\circ}{81}$					$\frac{4}{81}$	
NP			\mathbf{NP}					
\mathbf{NP}	PP		NP PP					
Det N P	NP		Det N					
the dog nea	\sim		the dog					
the dog het			the dog					
	the							
	<u>12</u>		_4_					
	81		27					

Our more complex examples

0.66	0.64	0.91	0.90	0.89	0.90	0.91	0.87	0.37
S	S	S	S	S	S	S	S	N/A
NP	۱ NP	NP VP	NP VP	NP VP	NP VP	NP VP	NP VP	
1		\sim 1	\sim \sim	\sim	\wedge \frown	\wedge \frown	\wedge \frown	
DT	DT NN	DT NN VBD	DT NN VBD NP	DT NN VBD NP	DT NN VBD NP PP	DT NN VBD NP PP	DTNN VBD NP PP	
			1	\sim	\sim .	~ ^	$\sim \sim$	
			DT	DTNN	DT NN IN	DT NN IN NI	P DT NN IN NP	
						1	\sim	
						D	T DT NI	V
0.15	0.19	0.04	0.04	0.04	0.05	0.03	0.05	0.63
S	S	S	S	S	S	S	S	S
`	`	`	1	1	١	1	١.	
NP	NP	NP	NP	NP	NP	NP	NP	NP VP
I	1							
NP	NP	NP VP	NP VP	NP VP	NP VP	NP VP	NP VP	NP VP VBD
I		\sim '	\sim	\sim	\wedge \frown	\wedge \frown	\sim \sim	\wedge \rightarrow
DT	DTNN	DT NN VBN	<i>DT NN VBN</i> NP	DT NN VBN NP	DT NN VBN NP PP	DT NN VBN NP PP	DTNN VBN NP PP	DT NN VBN NP PP
			1	\sim	\sim 1	~ ^	\sim	\sim
			DT	DT NN	DT NN IN	DT NN IN NE	DT NN IN NP	DT NN IN NP
						1	\sim	\sim
-					0	DI	DTNN	DT NN
The	woma	n brought	the	sandwich	from	the	kitchen	tripped

Ingredients for modeling human syntactic processing

• Estimate of statistics of the linguistic environment



Focus on predictive, incremental processing



An incremental probabilistic (Earley) parsing model

		1	De	t ightarrow the
$\frac{2}{3}$	$NP \to Det~N$	$\frac{2}{3}$	Ν	ightarrow dog
<u>1</u> 3	$NP \to NP \; PP$	<u>1</u> 3	Ν	ightarrow cat
ĭ	$PP \to P \; NP$	ĭ	Ρ	ightarrow near



• Let a word's difficulty be its *surprisal* given its context:

$$egin{aligned} ext{Surprisal}(w_i) &\equiv & \lograc{1}{P(w_i| ext{CONTEXT})} \ & \left[pprox & \lograc{1}{P(w_i|w_{1\cdots i-1})}
ight] \end{aligned}$$

(Hale, 2001, NAACL; Levy, 2008, Cognition)

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- Predictable words are read faster (Ehrlich & Rayner, 1981) and have distinctive EEG responses (Kutas & Hillyard 1980)
- Combine with probabilistic grammars to give grammatical (Hale, 2001, NAACL; Levy, 2008, Cognition)

The surprisal graph



When the dog scratched the vet and his new assistant removed the muzzle.

(Frazier & Rayner, 1982)

• Here's a *local syntactic ambiguity*

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• Compare with:



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S	ightarrow Sbar S	0.3	Conj	m i ightarrow and	1	Adj	ightarrow new	1
S	ightarrow NP VP	0.7	Det	ightarrow the	0.8	VP	ightarrow V NP	0.5
SBAR	ightarrow COMPL S	0.3	Det	\rightarrow its	0.1	VP	$\rightarrow V$	0.5
SBAR	ightarrow COMPL S COMMA	0.7	Det	ightarrow his	0.1	V	ightarrow scratched	0.25
COMPL	ightarrow When	1	Ν	ightarrow dog	0.2	V	ightarrow removed	0.25
NP	\rightarrow Det N	0.6	N	ightarrow vet	0.2	V	ightarrow arrived	0.5
NP	ightarrow Det Adj N	0.2	N	ightarrow assistant	0.2	СОММА	ightarrow ,	1
NP	ightarrow NP Conj NP	0.2	N	ightarrow muzzle	0.2			
			N	ightarrow owner	0.2			

S	ightarrow Sbar S	0.3	Con	m i ightarrow and	1	Adj	ightarrow new	1
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NP	ightarrow NP Conj NP	0.2	Ν	ightarrow muzzle	0.2			
			N	ightarrow owner	0.2			

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NP	ightarrow NP Conj NP	0.2	N	ightarrow muzzle	0.2			
			N	ightarrow owner	0.2			

S	\rightarrow	SBAR S	0.3
S	\rightarrow	NP VP	0.7
SBAR	\rightarrow	COMPL S	0.3
SBAR	\rightarrow	COMPL S COMMA	0.7
COMPL	\rightarrow	When	1
NP	\rightarrow	Det N	0.6
NP	\rightarrow	Det Adj N	0.2
NP	\rightarrow	NP Conj NP	0.2

	Conj	\rightarrow	and
	Det	\rightarrow	the
١	Det	\rightarrow	its
	Det	\rightarrow	his
	Ν	\rightarrow	dog
	Ν	\rightarrow	vet
	Ν	\rightarrow	assistant
J	Ν	\rightarrow	muzzle
	Ν	\rightarrow	owner

1	Adi	ightarrow new		1
0.8	VP	\rightarrow V N	Р	0.5
0.1	VP	ightarrow V		0.5
0.1	V	ightarrow scra	tched	0.25
0.2	V	ightarrow remo	oved	0.25
0.2	V	$\rightarrow \operatorname{arriv}$	'ed	0.5
0.2	COMMA	ightarrow ,		1
0.2				
0.2				

• "Garden-path" analysis:



(analysis in Levy, 2013) 84











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(analysis in Levy, 2013) 84

• "Garden-path" analysis:



Disambiguating word probability marginalizes over incremental trees:

"Garden-path" analysis:



Disambiguating word probability marginalizes over incremental trees:

$$P(\text{removed}|w_{1...10}) = \sum_{T} P(\text{removed}|T)P(T|w_{1...10})$$

= 0 × 0.826 + 0.25 × 0.174

84 (analysis in Levy, 2013)

Preceding context can disambiguate

• *"its owner"* takes up the object slot of *scratched*





Sensitivity to verb argument structure

• A superficially similar example:

When the dog arrived the vet and his new assistant removed the muzzle.


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Sensitivity to verb argument structure

• A superficially similar example:

When the dog arrived the vet and his new assistant removed the muzzle. But harder here!

(c.f. When the dog scratched the vet and his new assistant removed the muzzle.)



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S	ightarrow NP VP	0.7	Det	ightarrow the	0.8	VP	ightarrow V NP	0.5
SBAR	ightarrow COMPL S	0.3	Det	ightarrow its	0.1	VP	$\rightarrow V$	0.5
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NP	ightarrow Det Adj N	0.2	Ν	ightarrow assistant	0.2	СОММА	ightarrow ,	1
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 The "context-free" assumption doesn't preclude relaxing probabilistic locality:

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$VP o V \ NP$	0.5		VP	ightarrow Vtrans NP	0.45
$VP\toV$	0.5		VP	ightarrow Vtrans	0.05
V~ ightarrowscratched	0.25	Replaced by	VP	ightarrow Vintrans	0.45
V~ ightarrow removed	0.25	\Rightarrow	VP	ightarrow Vintrans NP	0.05
$V \rightarrow arrived$	0.5		Vtrans	ightarrow scratched	0.5
	·		Vtrans	ightarrow removed	0.5
			Vintrans	$s ightarrow { ext{arrived}}$	1

(Johnson, 1998; Klein & Manning, 2003)





When the dog scratched the vet and his new assistant removed the muzzle.

Transitivity-distinguishing PCFG						
Condition	Ambiguity onset	Resolution				
Intransitive (arrived)	2.11	3.20				
Transitive (scratched)	0.44	8.04				

Move to broad coverage

- Instead of the pedagogical grammar, a "broad-coverage" grammar from the parsed Brown corpus (11,984 rules)
- Relative-frequency estimation of rule probabilities ("vanilla" PCFG)
- (We'll discuss these estimation techniques next class)



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- Gibson 1998, 2000 (DLT): multiple and/or more distant dependencies are harder to process

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Processing

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(Levy, Fedorenko, Breen, & Gibson, 2012) ⁹¹

 Equipped with a theory of probabilistic expectations, let's revisit more "memory"-oriented results

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easv

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The chair consulted the executive **of** the companies who was making lots of \$ The chair consulted the executive **about** the companies who was making lots of \$

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Probability & extraposition

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 Alternative hypothesis: processing extraposed RCs is hard because they're unexpected

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Levy, Fedorenko, Breen, & Gibson (2012)

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Expect Extr low – ... the executives about the company which was making... low + ... the executives about the company who were making... high – ... only those executives about the company which was making... high + ... only those executives about the company who were making...

 Our prediction is an *interactive effect*: high RC expectation ("only those") will facilitate RC reading, but *only* in the extraposed condition

- We crossed RC expectation (low/high) with RC extraposition (extraposed/unextraposed)
- Example sentence: The chairman consulted...

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high + ... only those executives about the company who were making...

- Our prediction is an *interactive effect*: high RC expectation ("only those") will facilitate RC reading, but *only* in the extraposed condition
- We tested this in a self-paced reading study

Levy, Fedorenko, Breen, & Gibson (2012)

Online processing results

• The difficulty pattern emerges within the RC's first 4 words:



Online processing results

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Online processing results

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Levy, Fedorenko, Breen, & Gibson (2012)

Expectations versus memory

- Suppose you know that some event class X has to happen in the future, but you don't know:
 - 1. When X is going to occur
 - 2. Which member of X it's going to be
- The things W you see before X can give you hints about (1) and (2)
 - If expectations facilitate processing, then seeing W should generally speed processing of X
- But you also have to keep W in memory and retrieve it at X
 - This could slow processing at X

 Variation in pre-verbal dependency structure also found in verb-final clauses such as in German

Die Einsicht, dass der Freund The insight, that the.NOM friend

dem Kunden das Auto aus Plastik the.DAT client the.ACC car of plastic

verkaufte, erheiterte die Anderen. sold, amused the others.

 Variation in pre-verbal dependency structure also found in verb-final clauses such as in German

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...daß der Freund DEM Kunden das Auto verkaufte ...that the friend the client the car sold







'...that the friend sold the client a car...'



...daß der Freund DES Kunden das Auto verkaufte ...that the friend the client the car sold '...that the friend of the client sold a car...'




What happens in German final-verb processing?



(Konieczny & Döring 2003)

What happens in German final-verb processing?



Locality: final verb read faster in *DES* condition Observed: final verb read faster in *DEM* condition

(Konieczny & Döring 2003)

daß



















Next: NPnom NPacc NPdat PP ADVP Verb



Next: NPnom NPacc NPdat PP ADVP Verb

























Model results

	Reading P(w _i): word time (ms) probability		Locality-based predictions
<i>dem Kunden</i> (dative)	555	8.38×10-8	slower
<i>des Kunden</i> (genitive)	793	6.35×10 ⁻⁸	faster

~30% greater expectation in dative condition once again, wrong monotonicity

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Back-pocket slides beyond here

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References

- Saenger spaces between words
- Kucera & Francis 1967
- Brown et al. 1990
- Garside et al. 1987
- Marcus et al. 1993