Neural networks for natural language

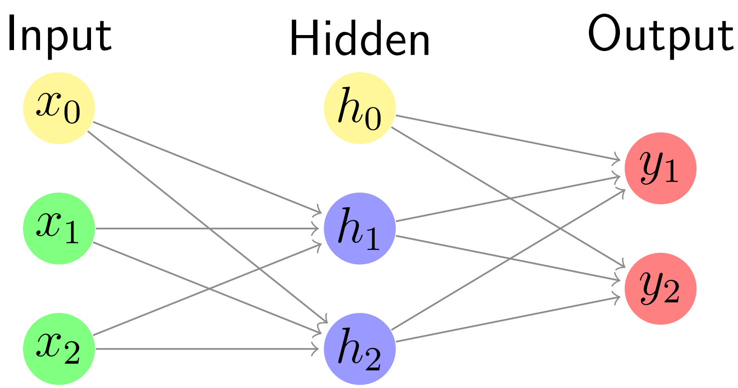
Roger Levy

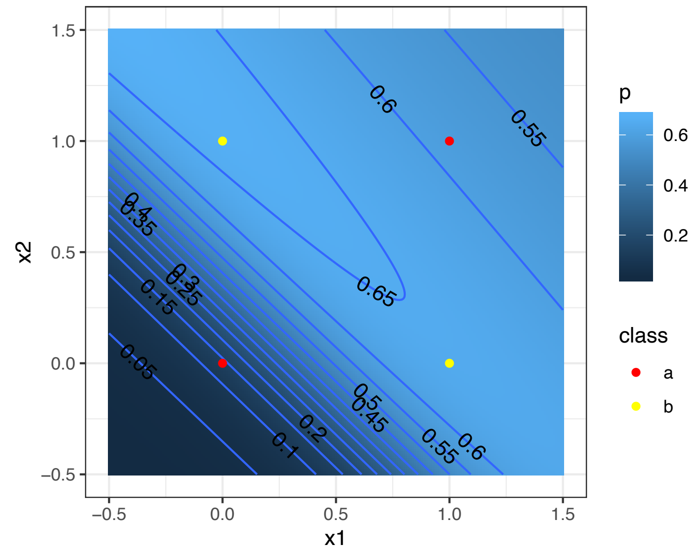
9.19: Computational Psycholinguistics

2 November 2023

Agenda for the day

- Last time: with a hidden layer, a NN can learn XOR...





- ...but language isn't just 2D input+2-class output! So, **today:**
- Dealing with language in neural networks
- Recurrent neural networks (RNNs)
  - Simple recurrent networks (SRNs)
  - Gated recurrent units (GRUs)
  - Long short-term memory networks (LSTMs)
- Examining RNN behavior

Dealing with language inputs

`Adam adores zebras ...`

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For language, input $\{x_i\}$ and output prediction y seem discrete:

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Simplest approach is ***localist*** or ***one-hot*** representations:

$$\texttt{Adam} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad \texttt{adores} \rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad \texttt{zebras} \rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Dealing with language inputs

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But lower-dimensional ***embeddings*** capture word similarities:

$$\texttt{Adam} \rightarrow \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix} \quad \texttt{adores} \rightarrow \begin{bmatrix} -0.3 \\ 0.4 \end{bmatrix} \quad \texttt{zebras} \rightarrow \begin{bmatrix} 0.7 \\ -0.1 \end{bmatrix}$$

Example feed-forward+embedding LM

Bengio et al., 2003: Neural *n*-gram language model

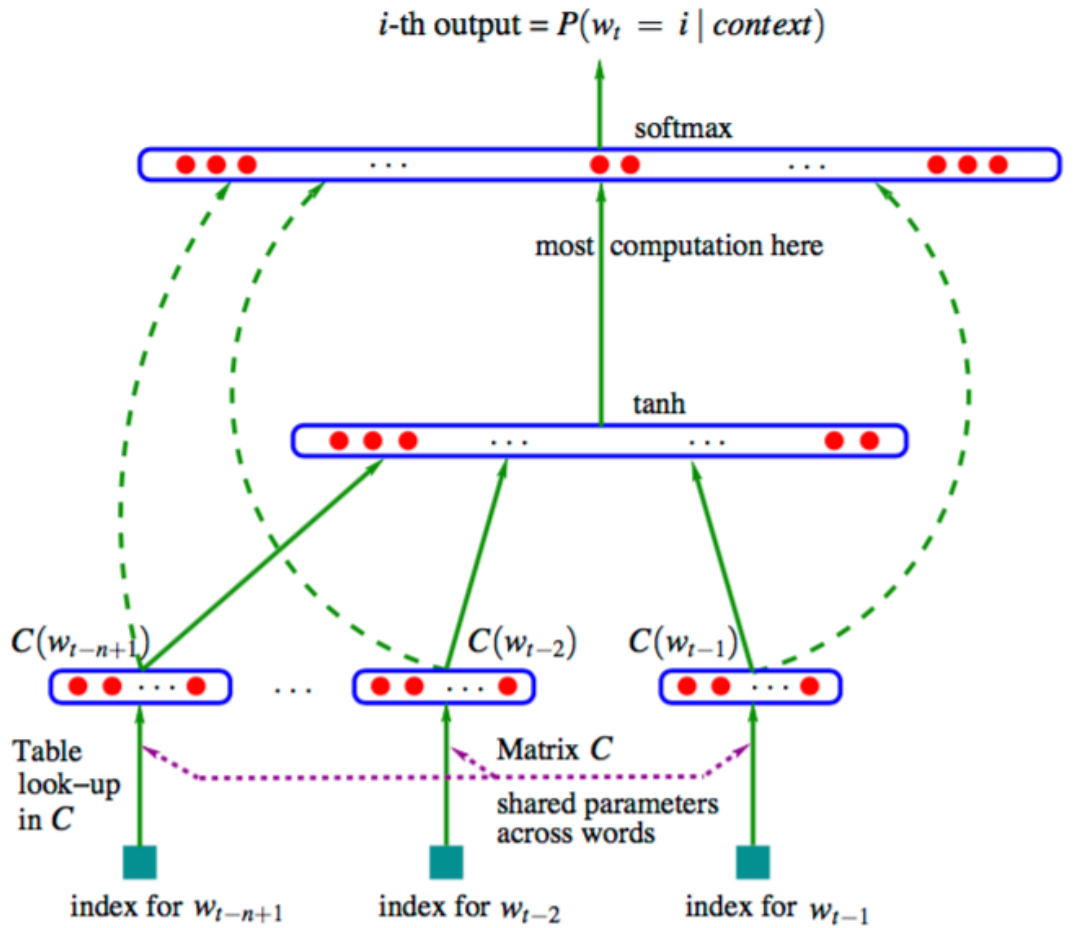


Figure 1: Neural architecture: $f(i, w_{t-1}, \cdots, w_{t-n+1}) = g(i, C(w_{t-1}), \cdots, C(w_{t-n+1}))$ where g is the neural network and $C(i)$ is the i-th word feature vector.

Old (2003!) perplexity results on Brown corpus

neural language models

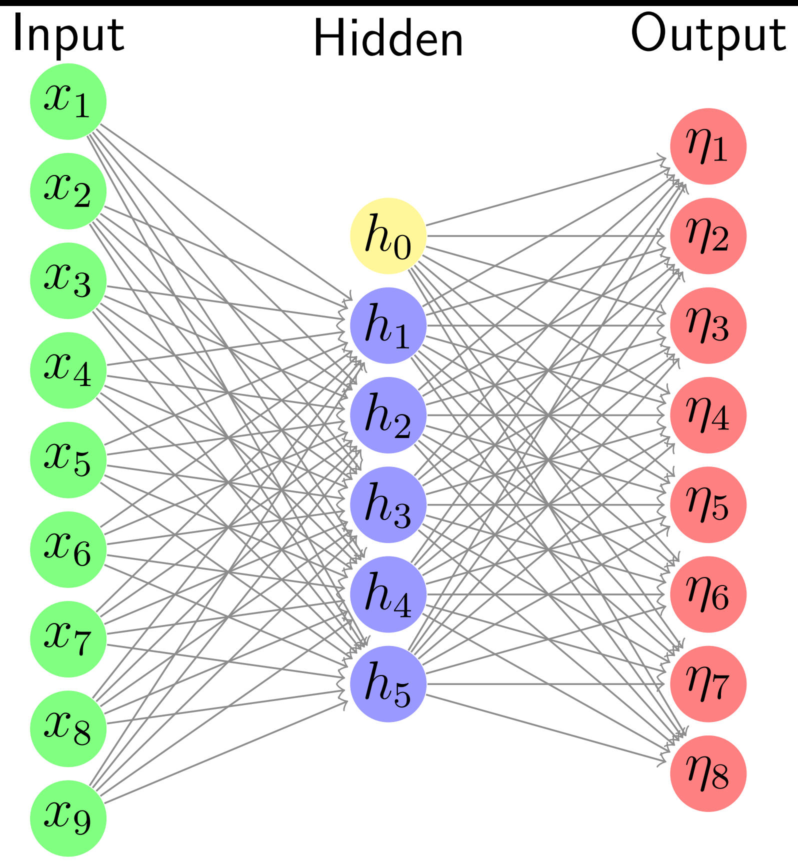
	n	c	h	m	direct	mix	train.	valid.	test.
MLP1	5		50	60	yes	no	182	284	268
MLP2	5		50	60	yes	yes		275	257
MLP3	5		0	60	yes	no	201	327	310
MLP4	5		0	60	yes	yes		286	272
MLP5	5		50	30	yes	no	209	296	279
MLP6	5		50	30	yes	yes		273	259
MLP7	3		50	30	yes	no	210	309	293
MLP8	3		50	30	yes	yes		284	270
MLP9	5		100	30	no	no	175	280	276
MLP10	5		100	30	no	yes		265	**252**

n-gram language models

	n	c	h	m	direct	mix	train.	valid.	test.
Del. Int.	3						31	352	336
Kneser-Ney back-off	3							334	323
Kneser-Ney back-off	4							332	321
Kneser-Ney back-off	5							332	321
class-based back-off	3	150						348	334
class-based back-off	3	200						354	340
class-based back-off	3	500						326	**312**
class-based back-off	3	1000						335	319
class-based back-off	3	2000						343	326
class-based back-off	4	500						327	312
class-based back-off	5	500						327	312

(Bengio et al., 2003)

The neural *n*-gram model



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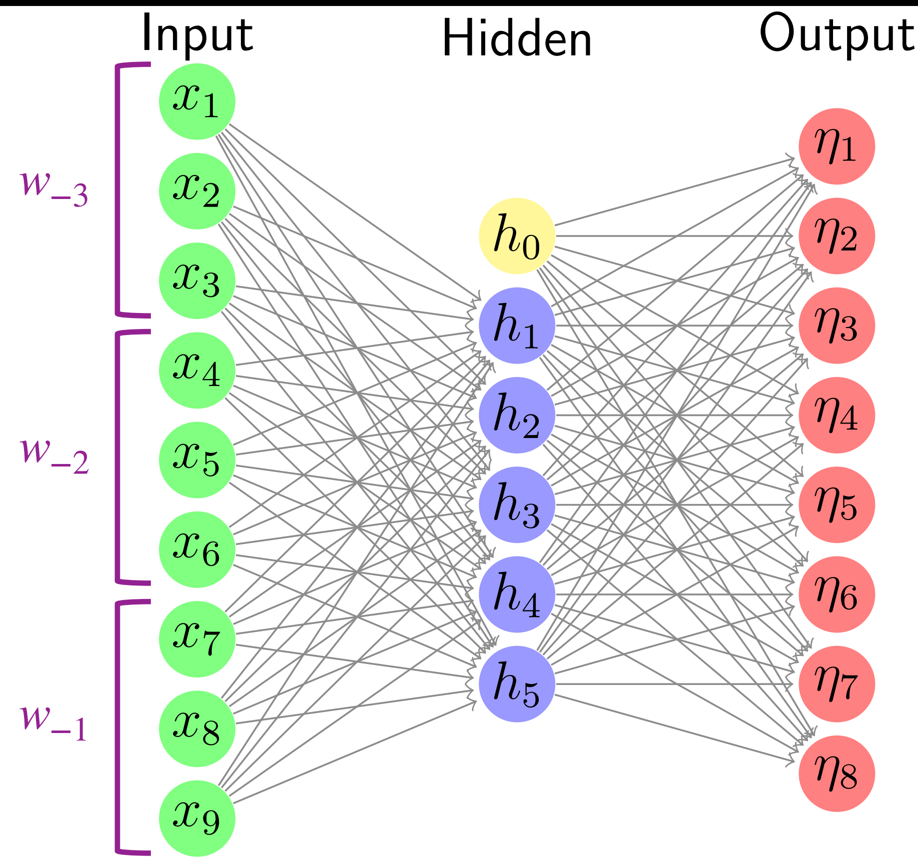

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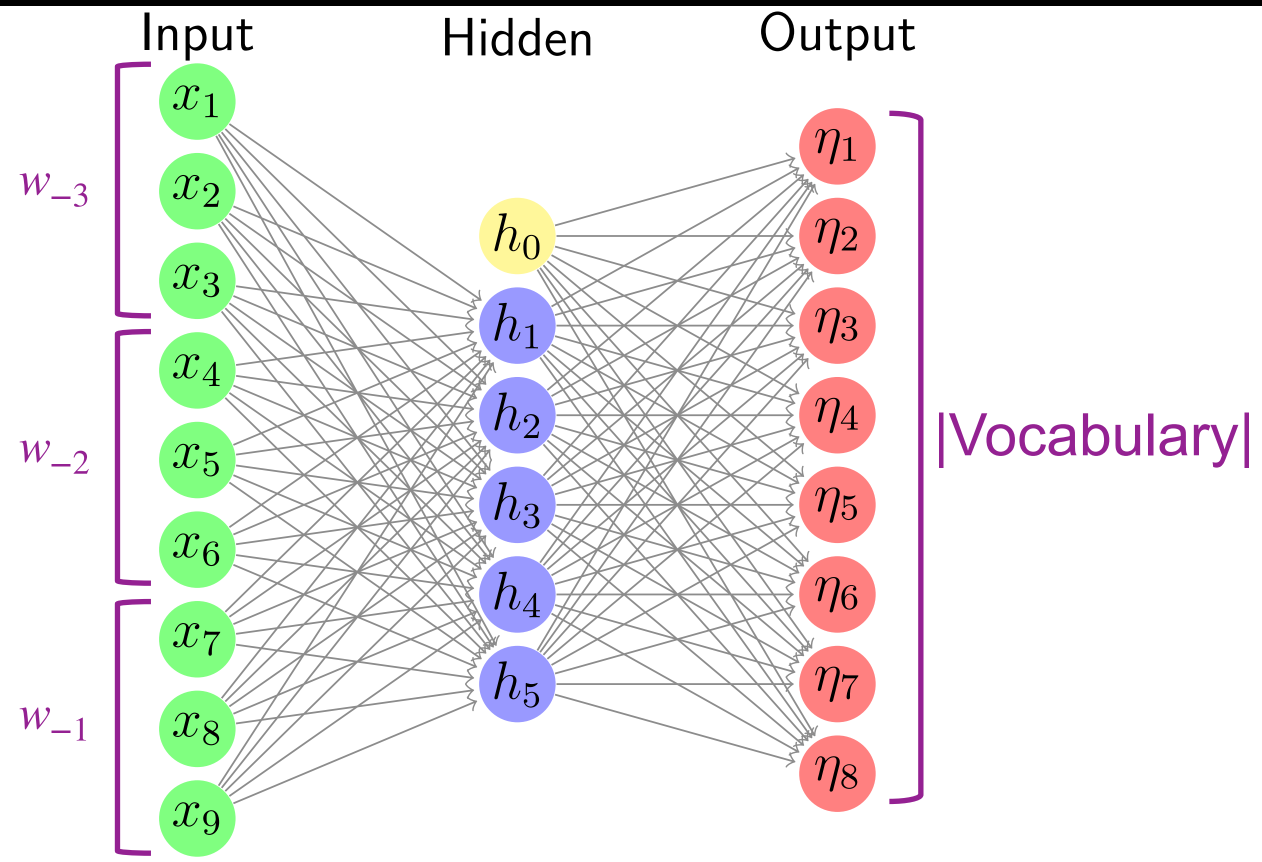

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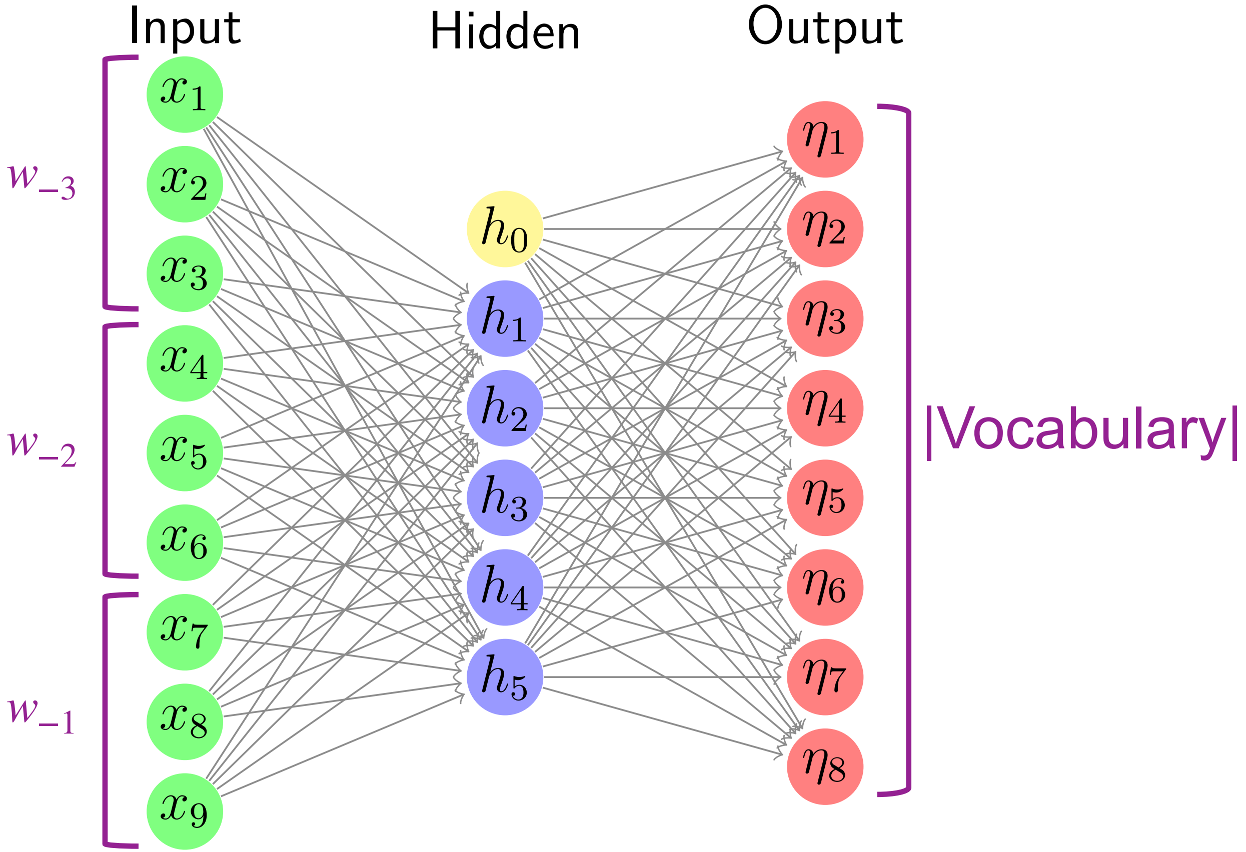
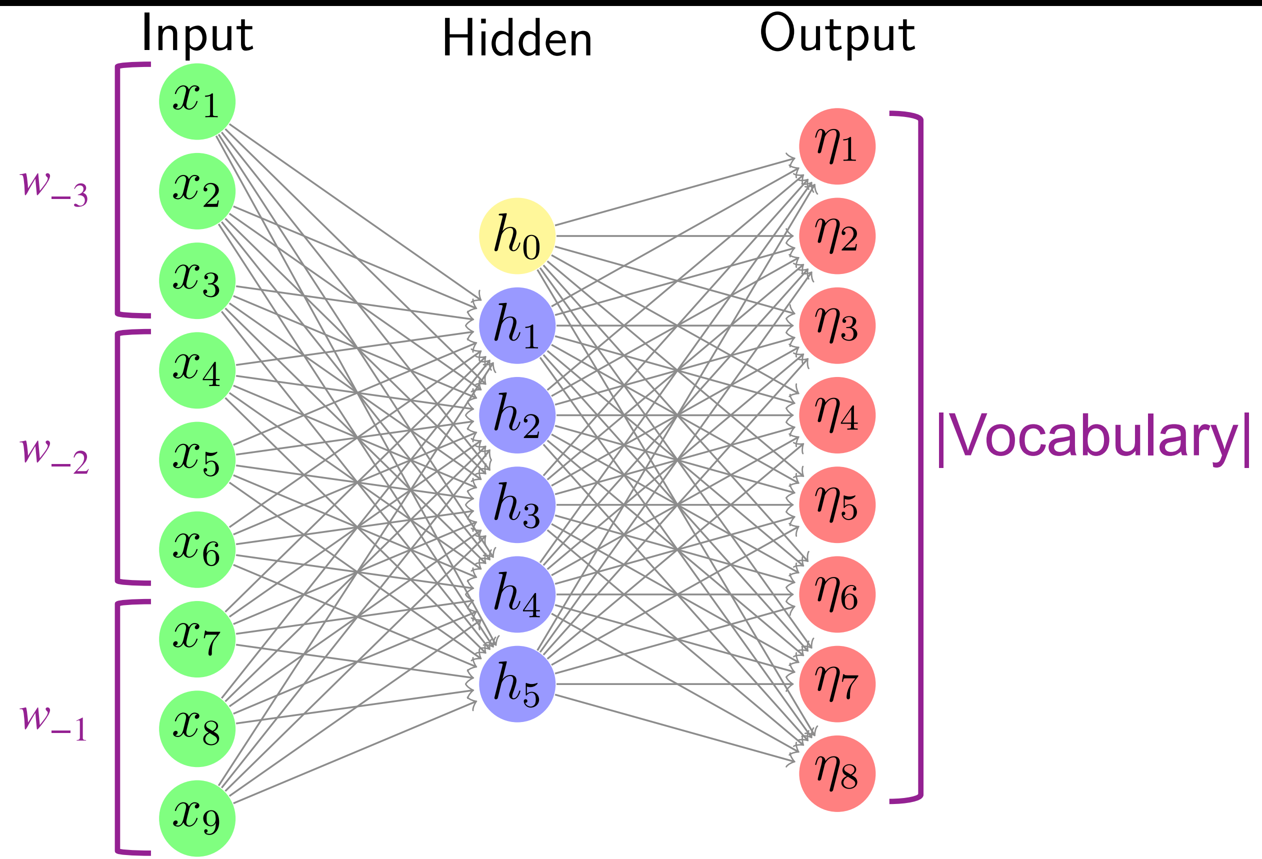


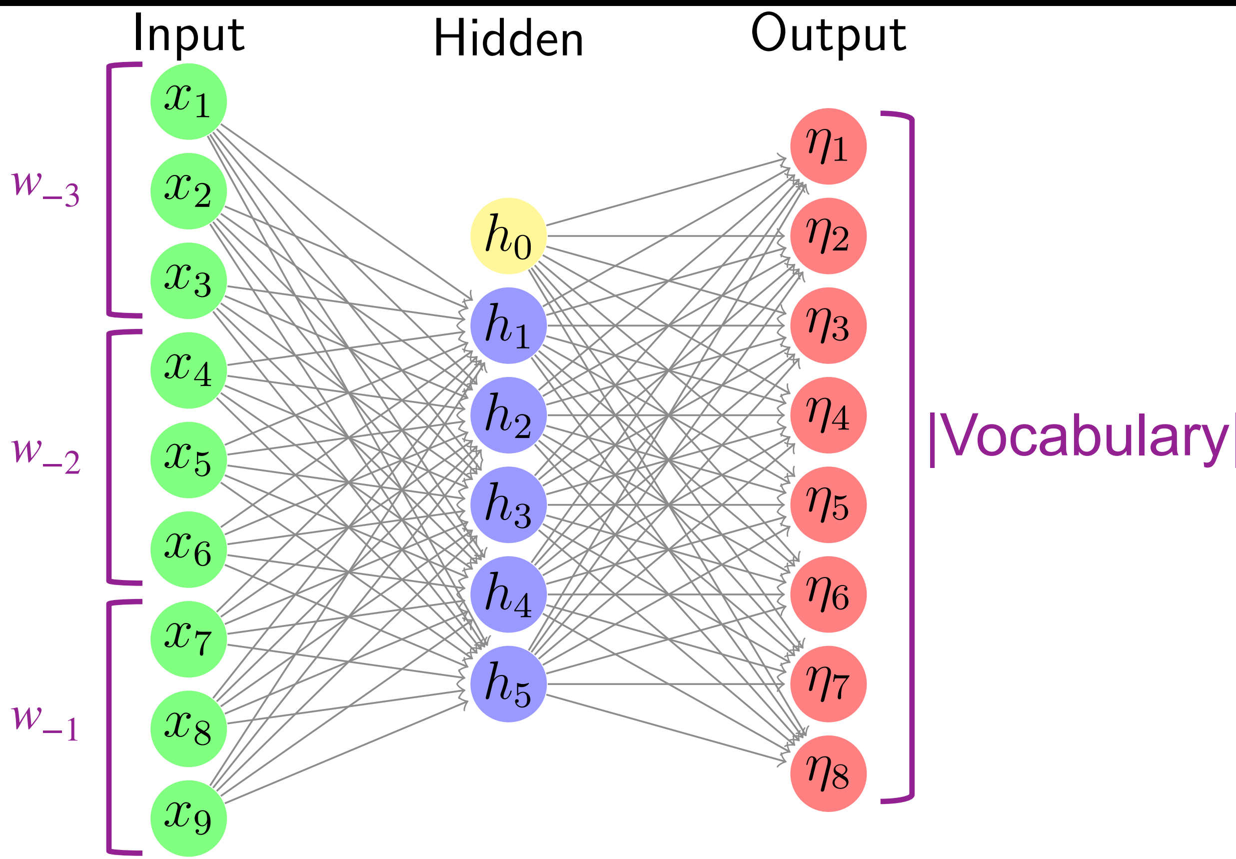
- **Advantages:** generalizes over *n*-gram contexts

The neural *n*-gram model



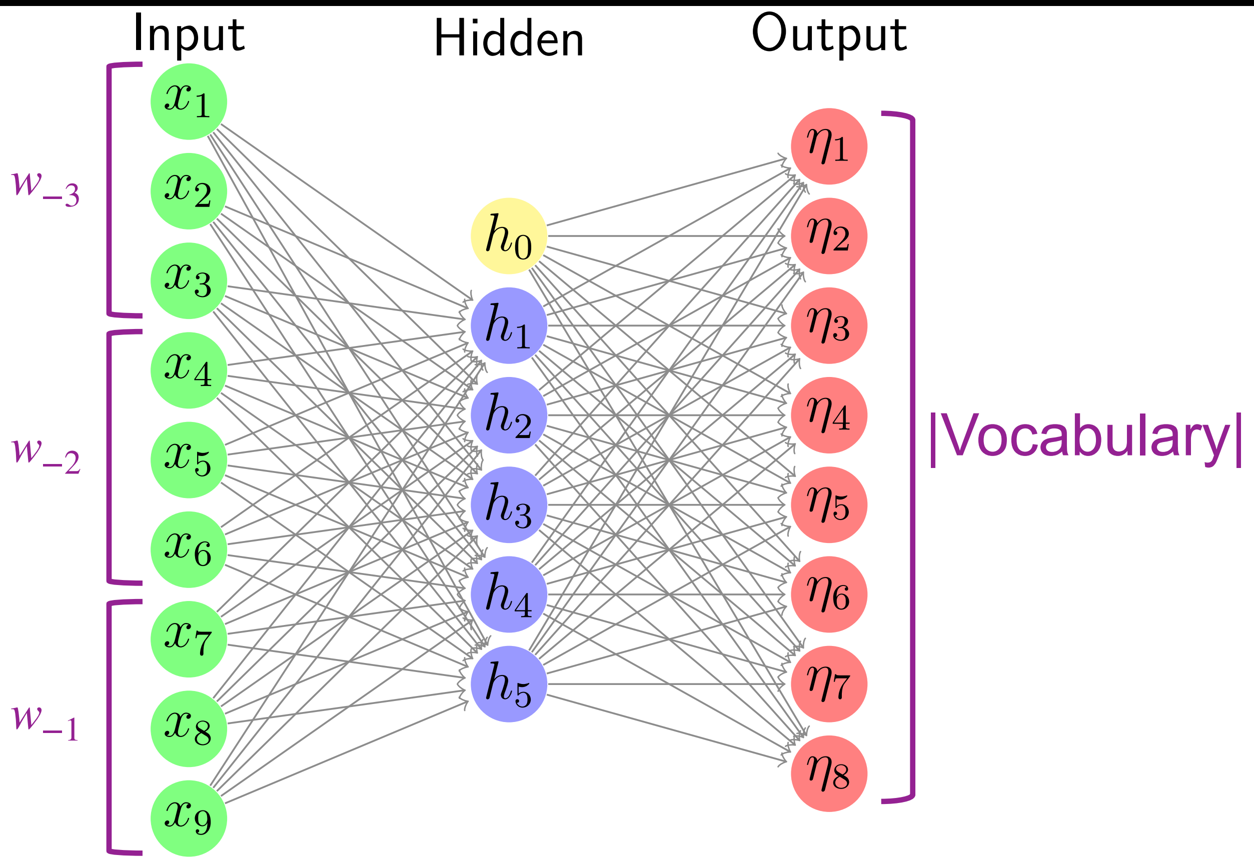
- **Advantages:** generalizes over *n*-gram contexts
- **Limitations:**

The neural *n*-gram model



- **Advantages:** generalizes over *n*-gram contexts
- **Limitations:**
  - this is for a fixed dimensionality input context

The neural *n*-gram model



- **Advantages:** generalizes over *n*-gram contexts
- **Limitations:**
  - this is for a fixed dimensionality input context
  - how to model variable-length context, like sentences?

Recurrent neural networks for language

(Jordan, 1986; Elman, 1990)

Recurrent neural networks for language

- Draw inspiration from real-time nature of human language processing

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The woman brought the sandwich from …

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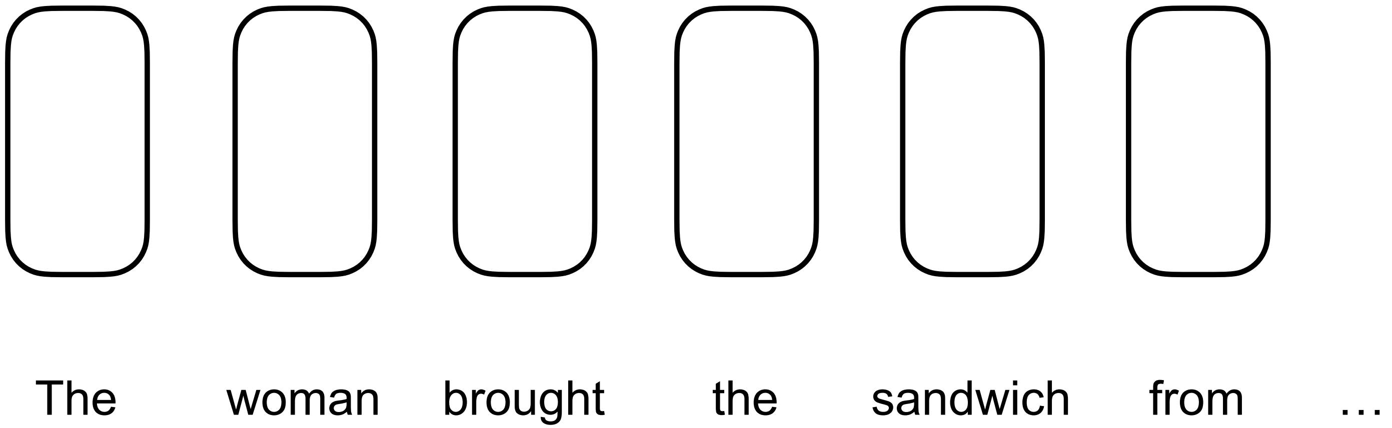

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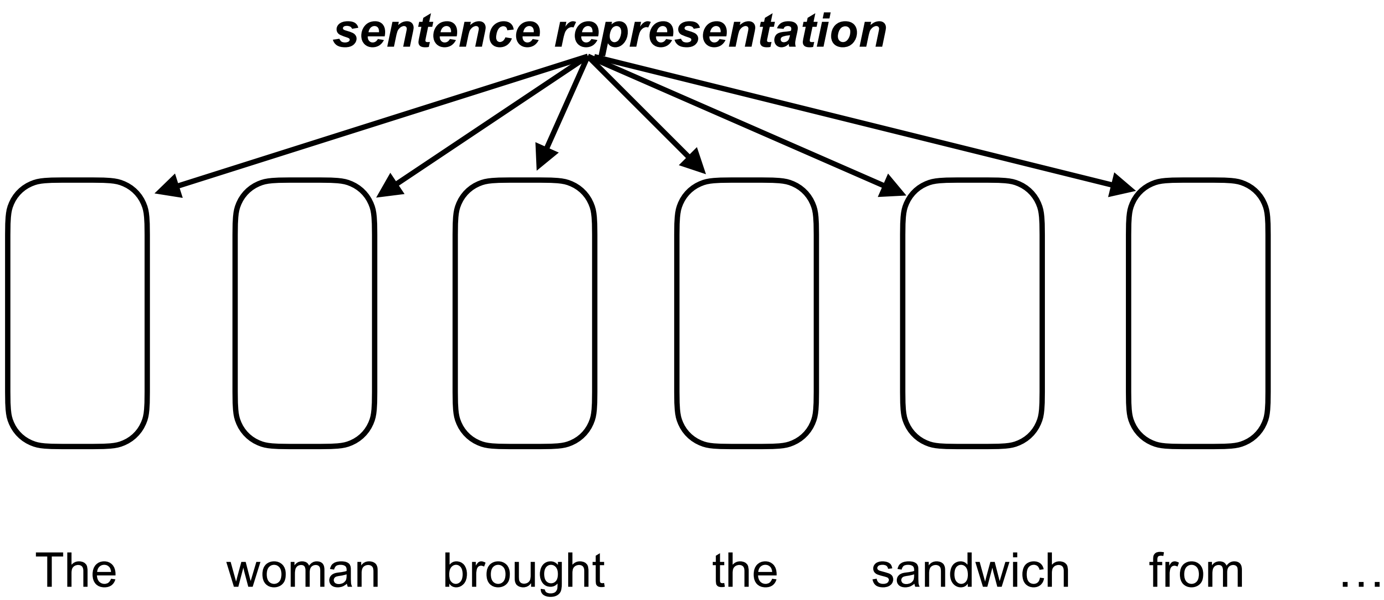

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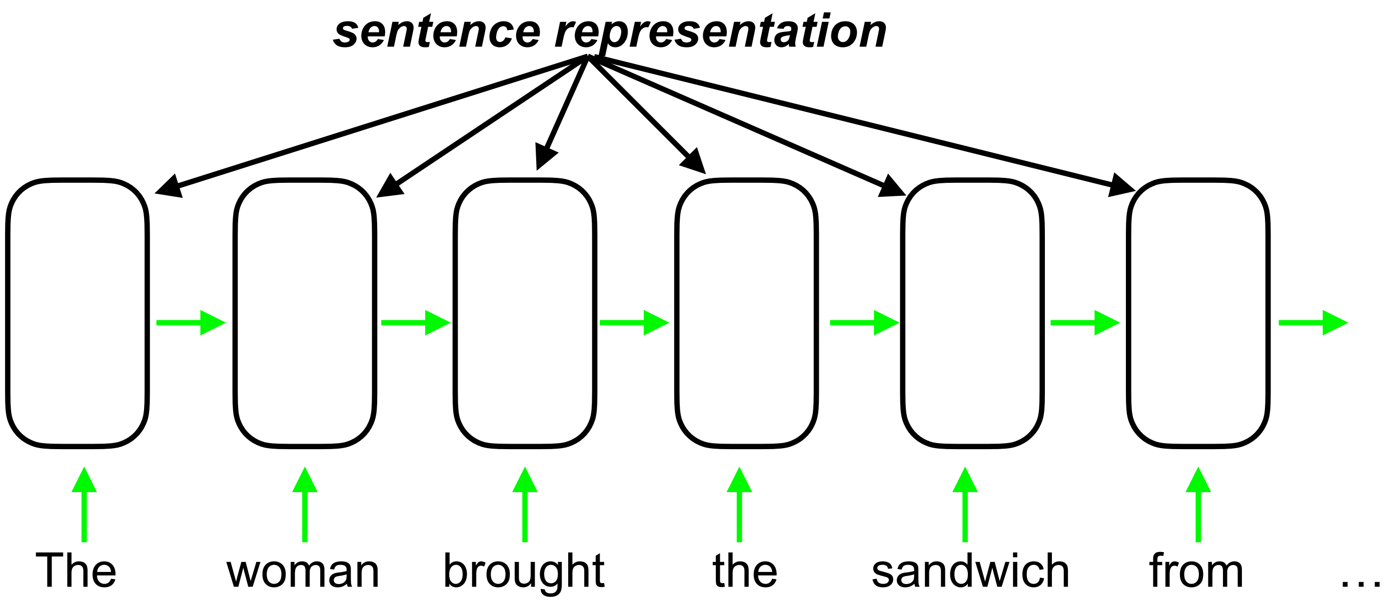
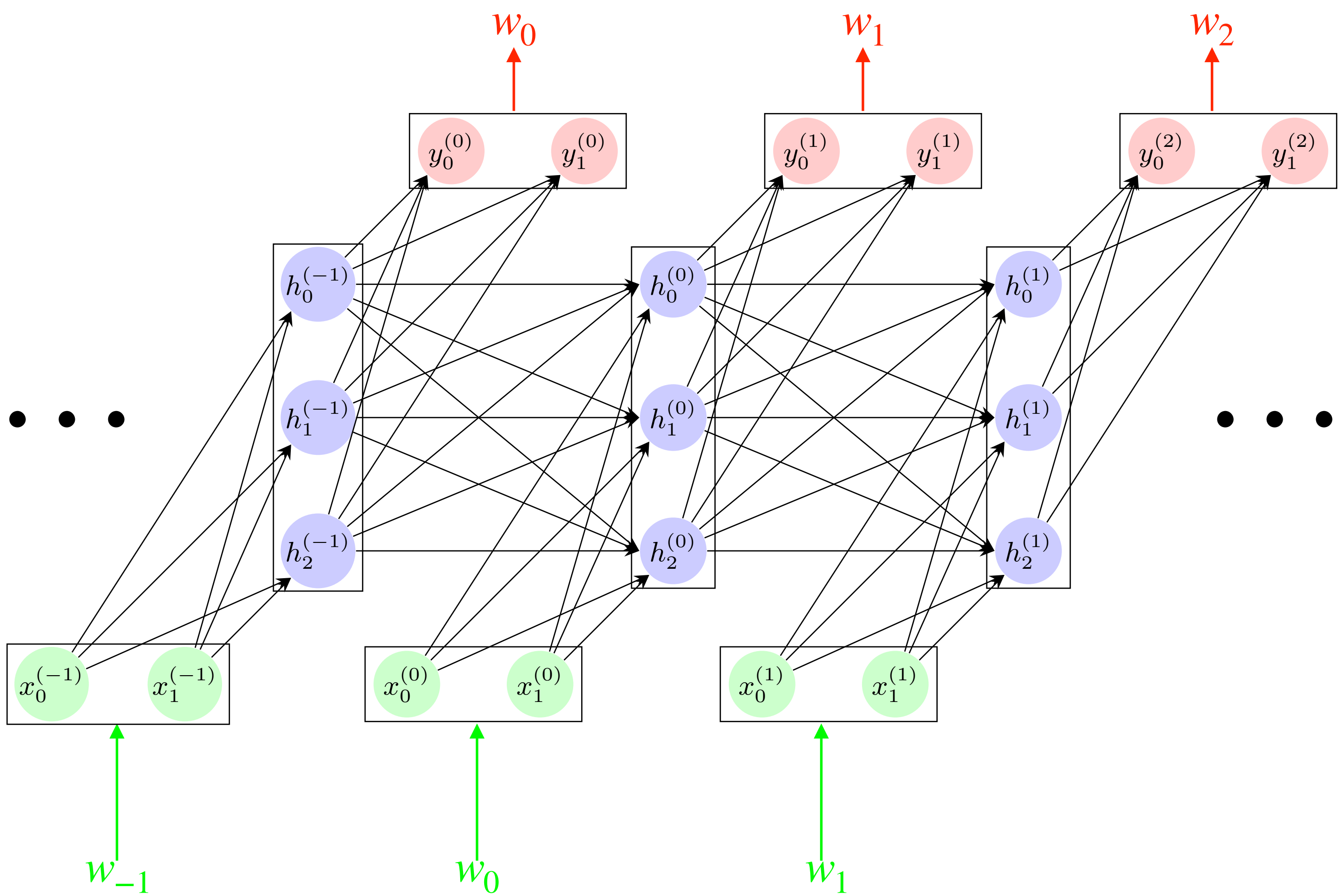


(Jordan, 1986; Elman, 1990)

The Simple Recurrent Network (SRN)

(bias nodes not shown)



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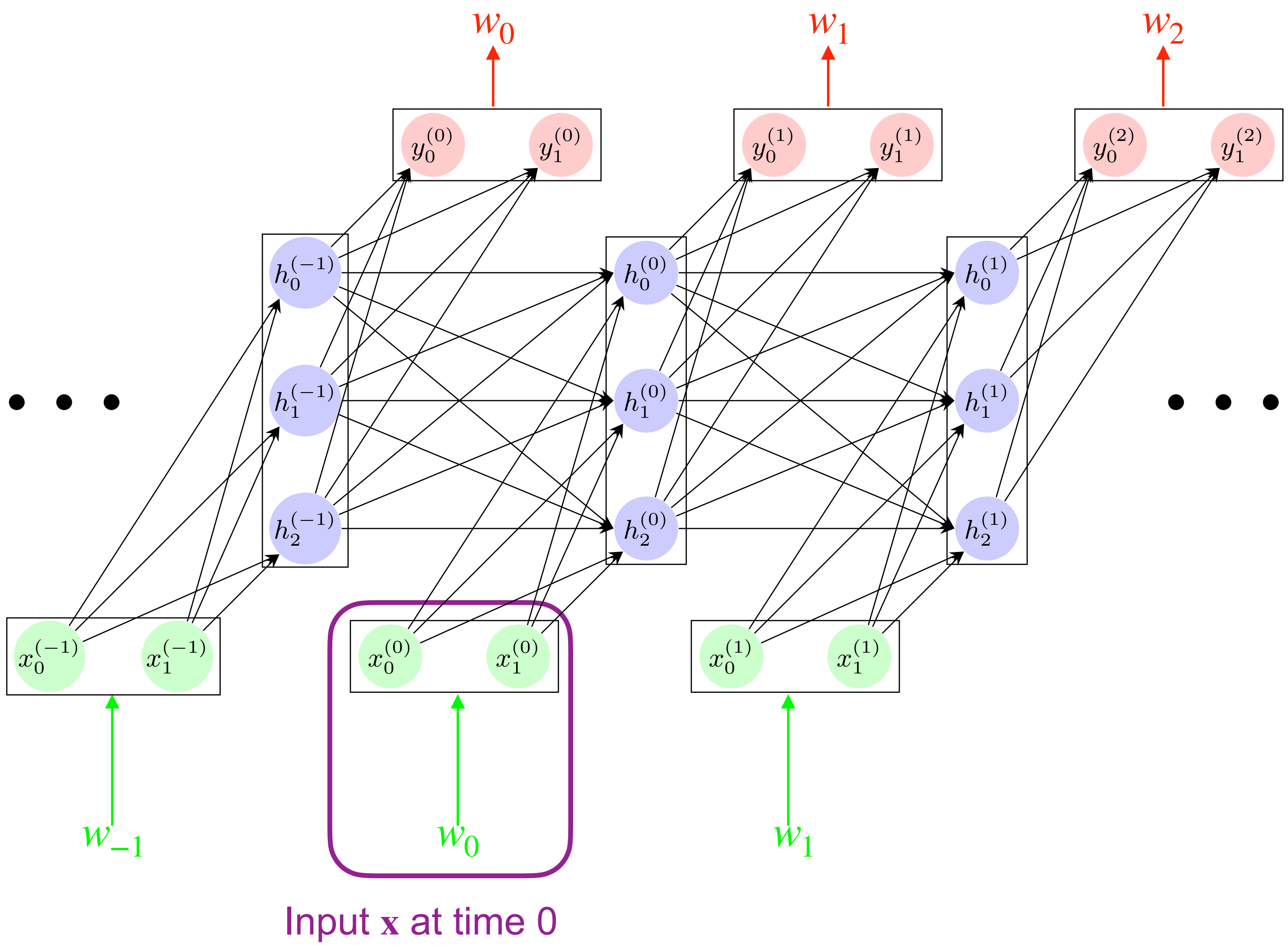

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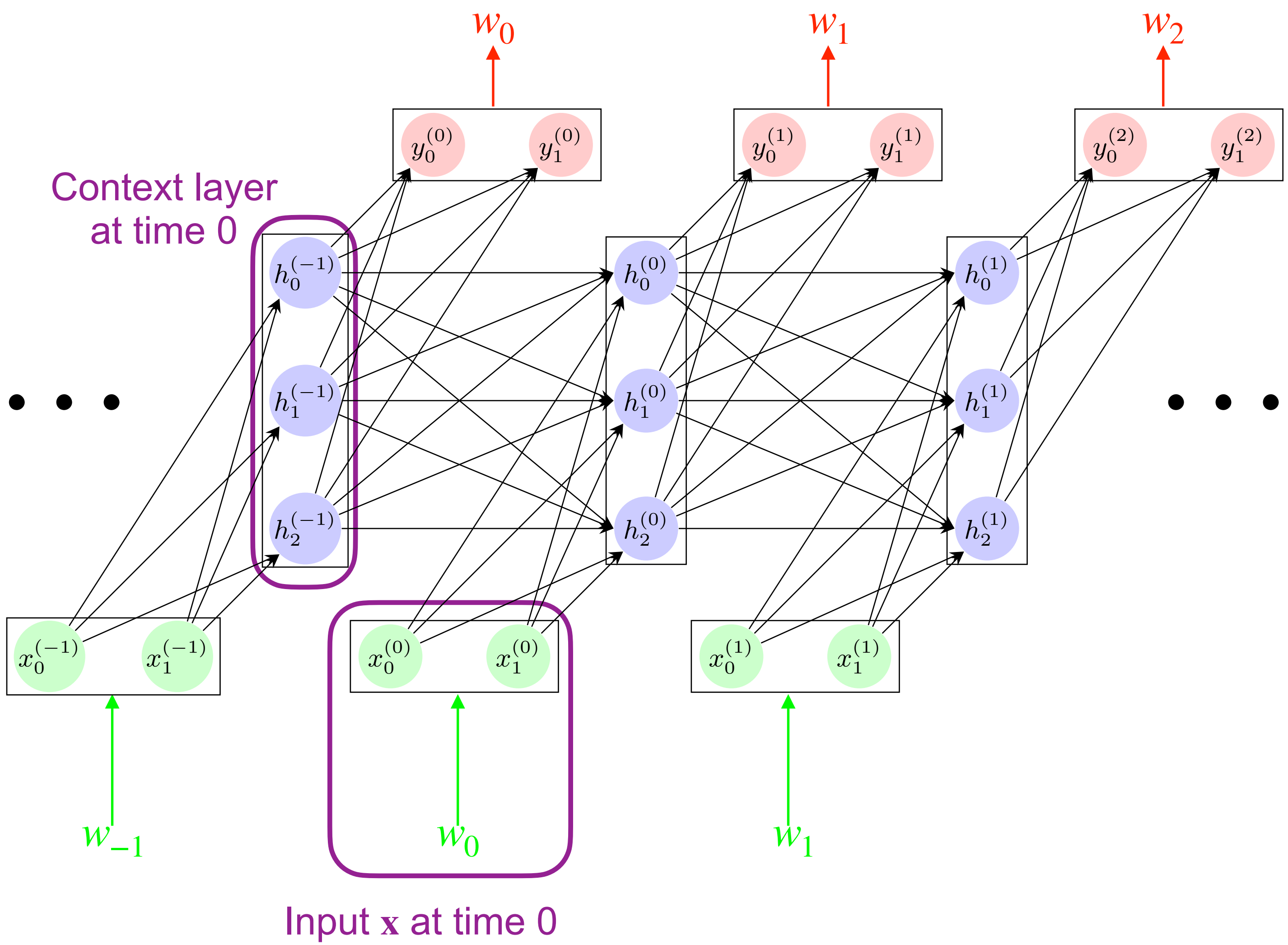

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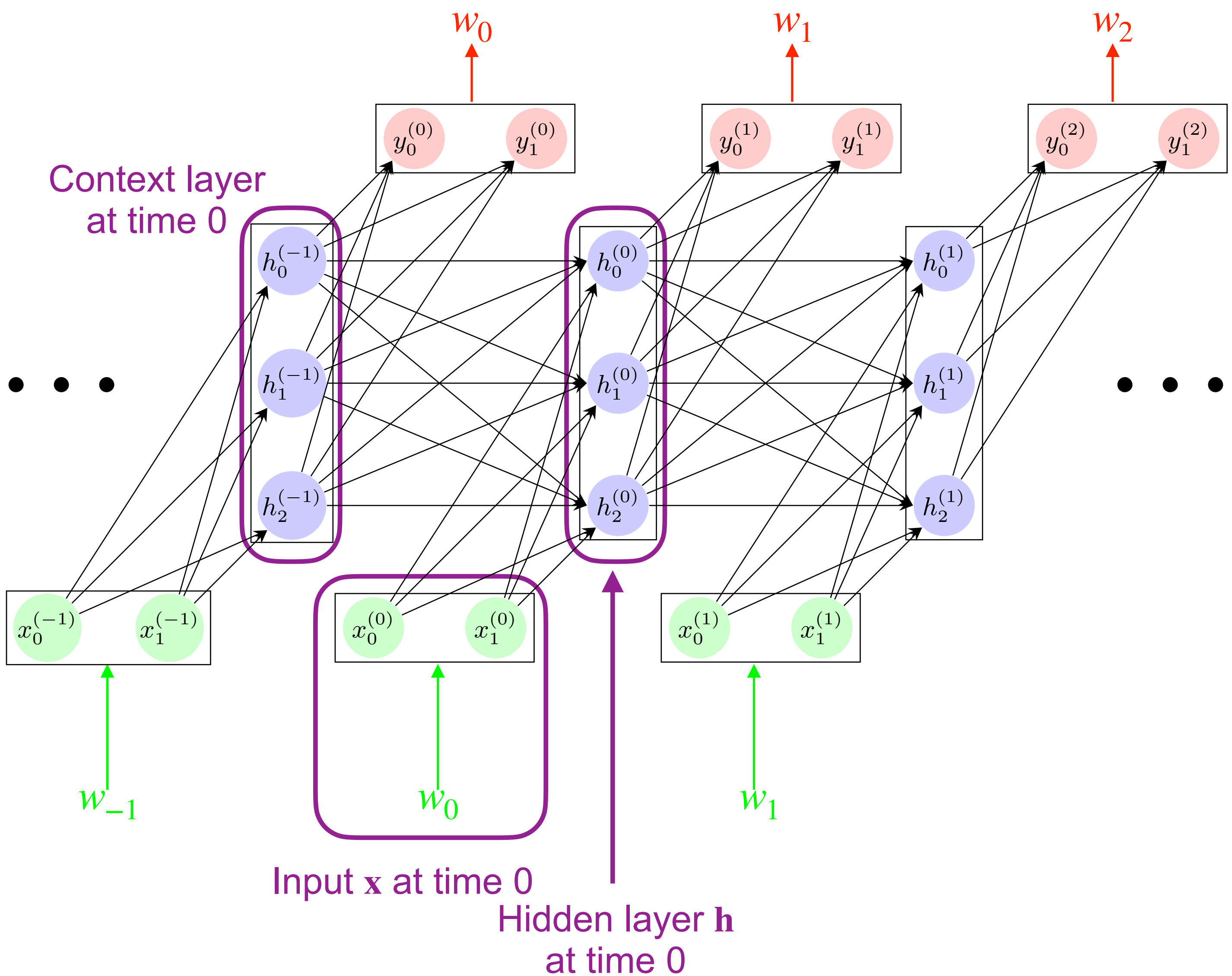

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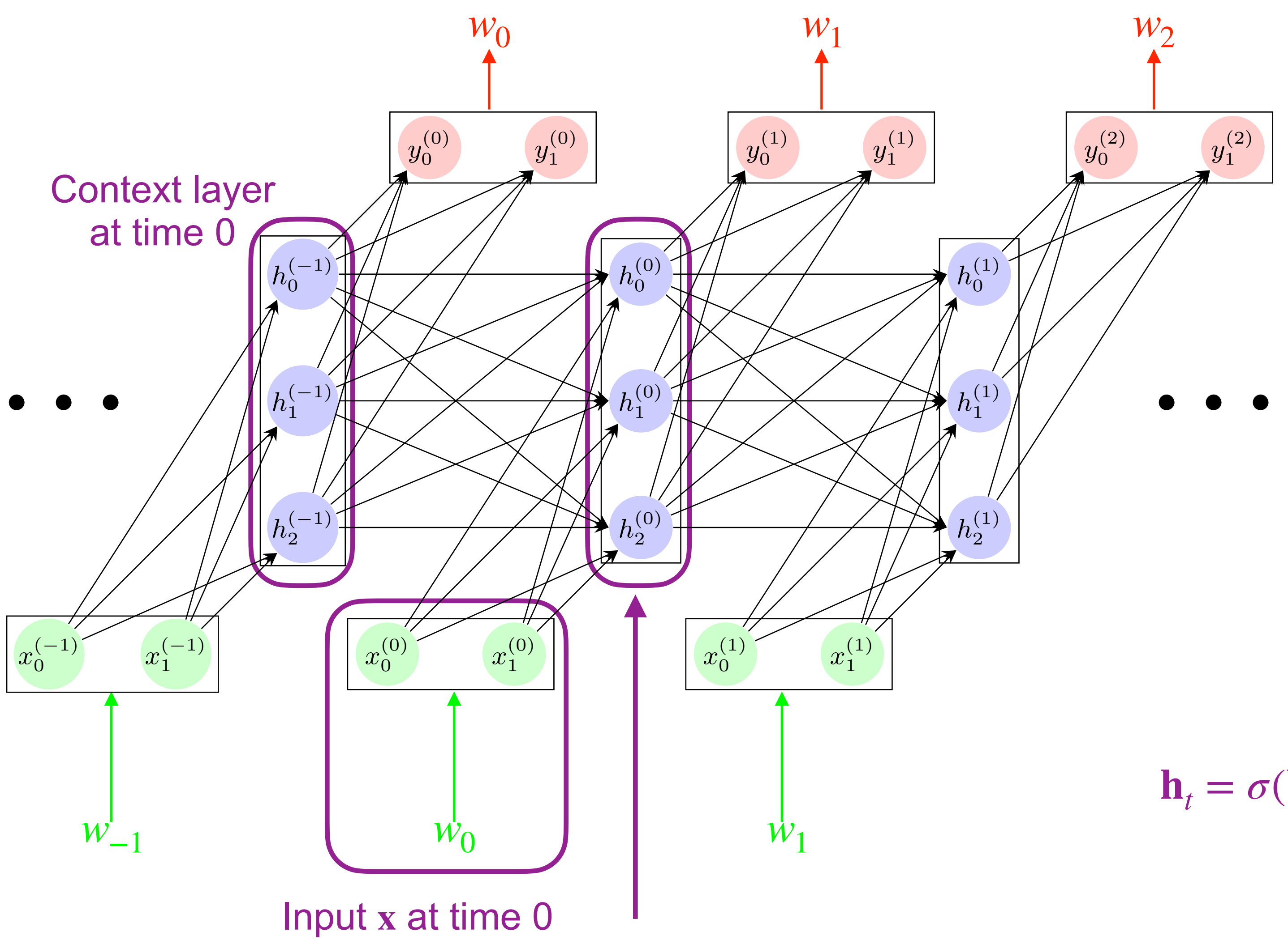

$$\mathbf{h}_t = \sigma(\mathbf{U_h}\mathbf{h}_{t-1} + \mathbf{W_h}\mathbf{x}_t)$$

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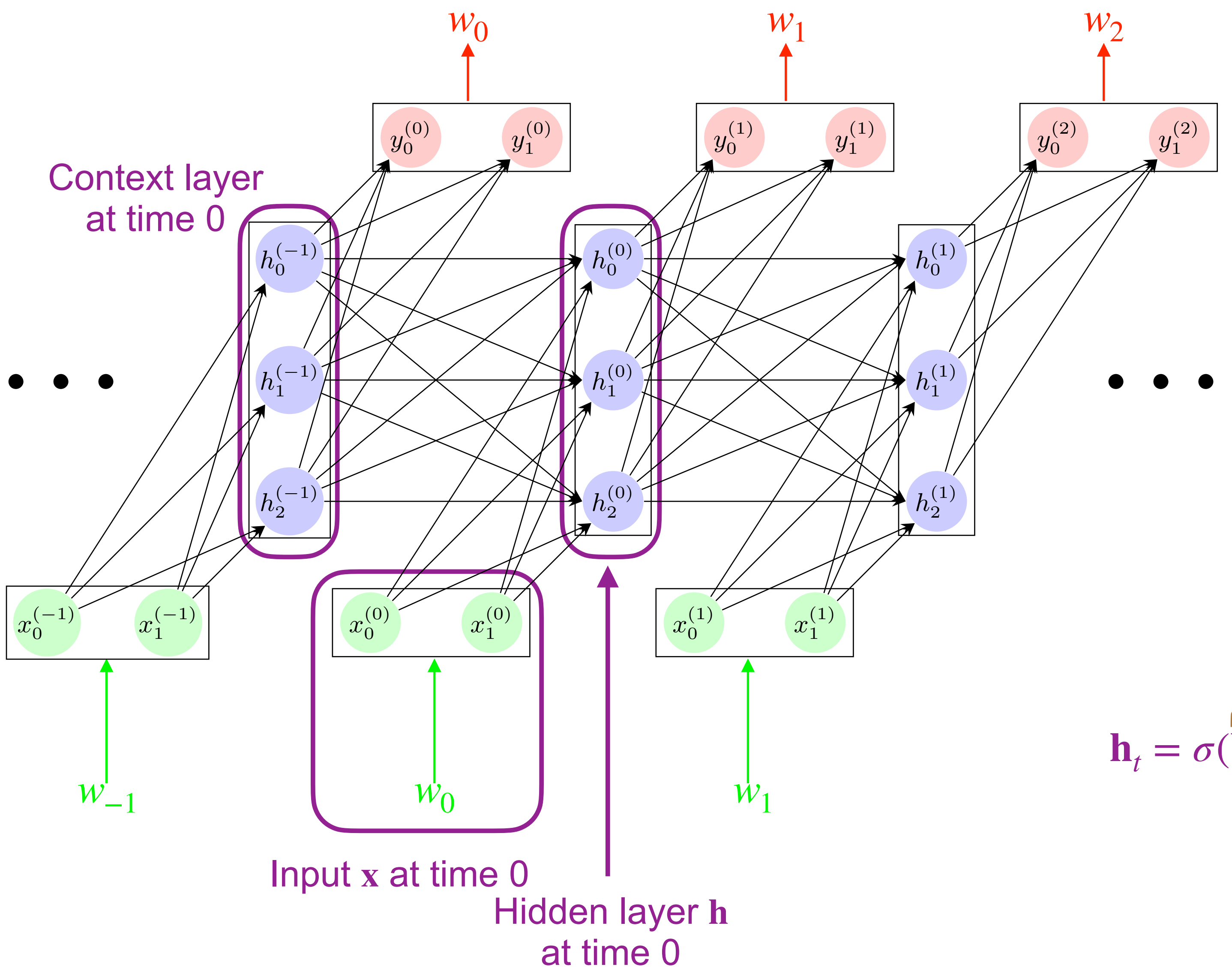

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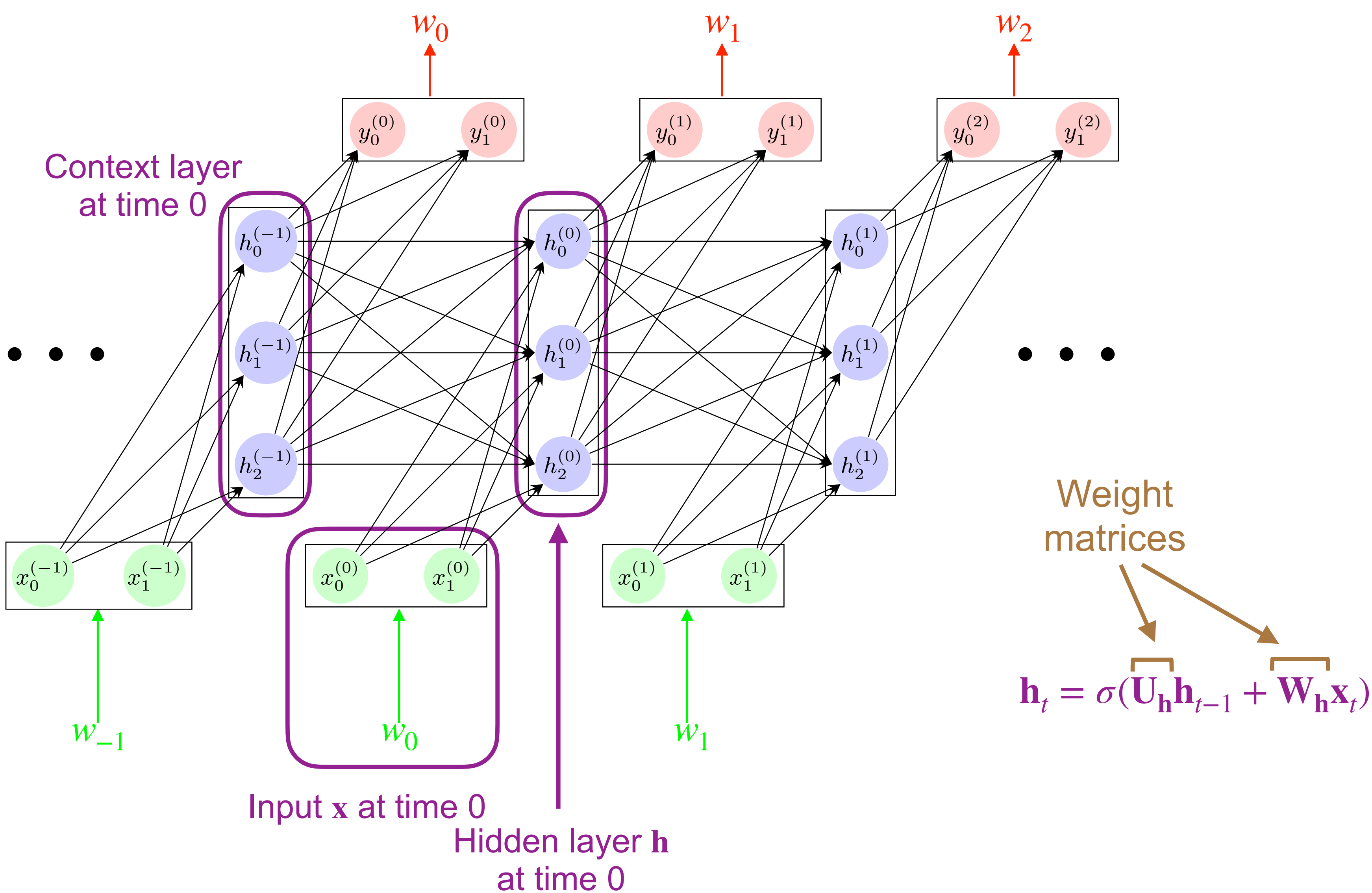

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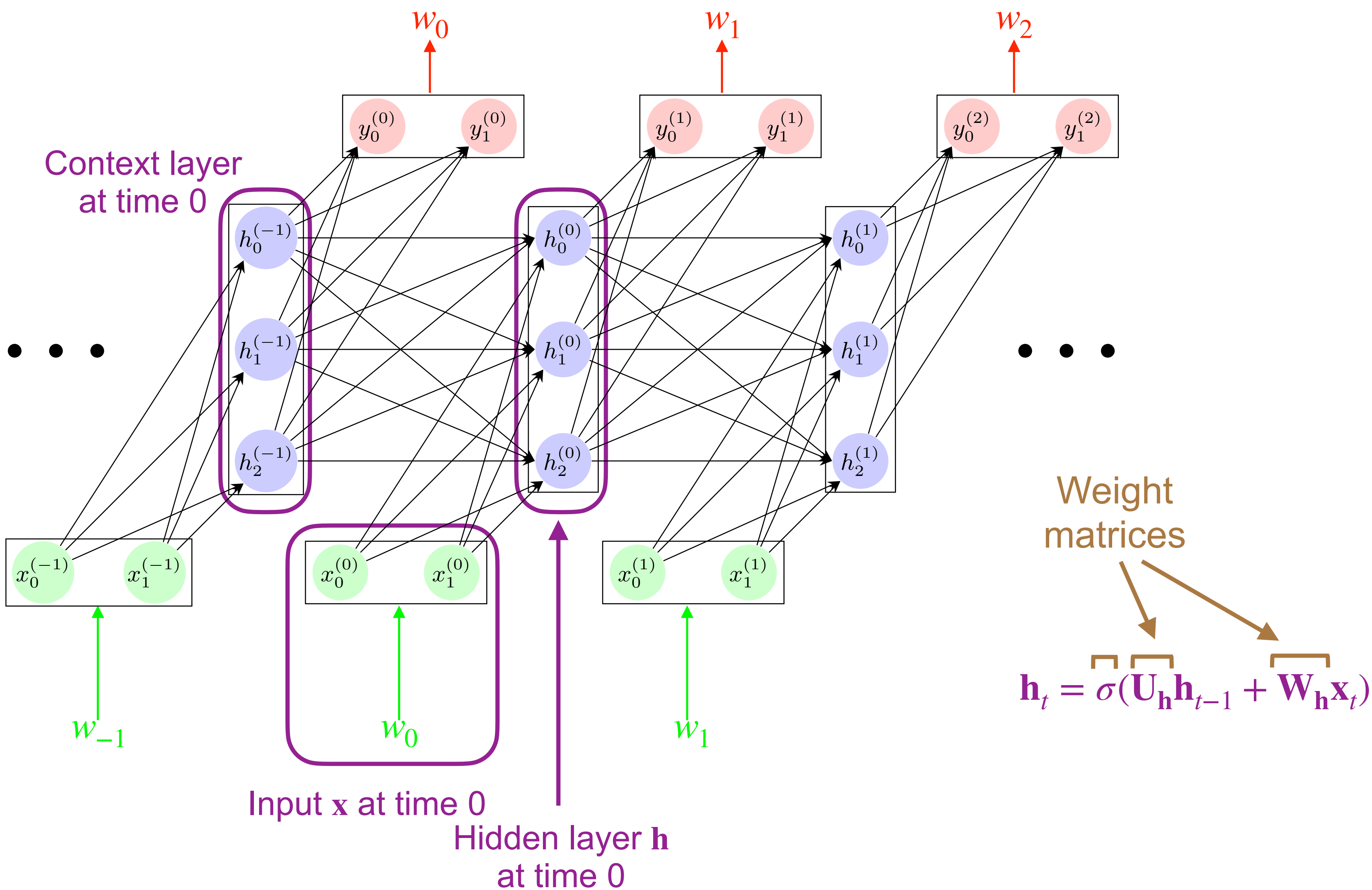

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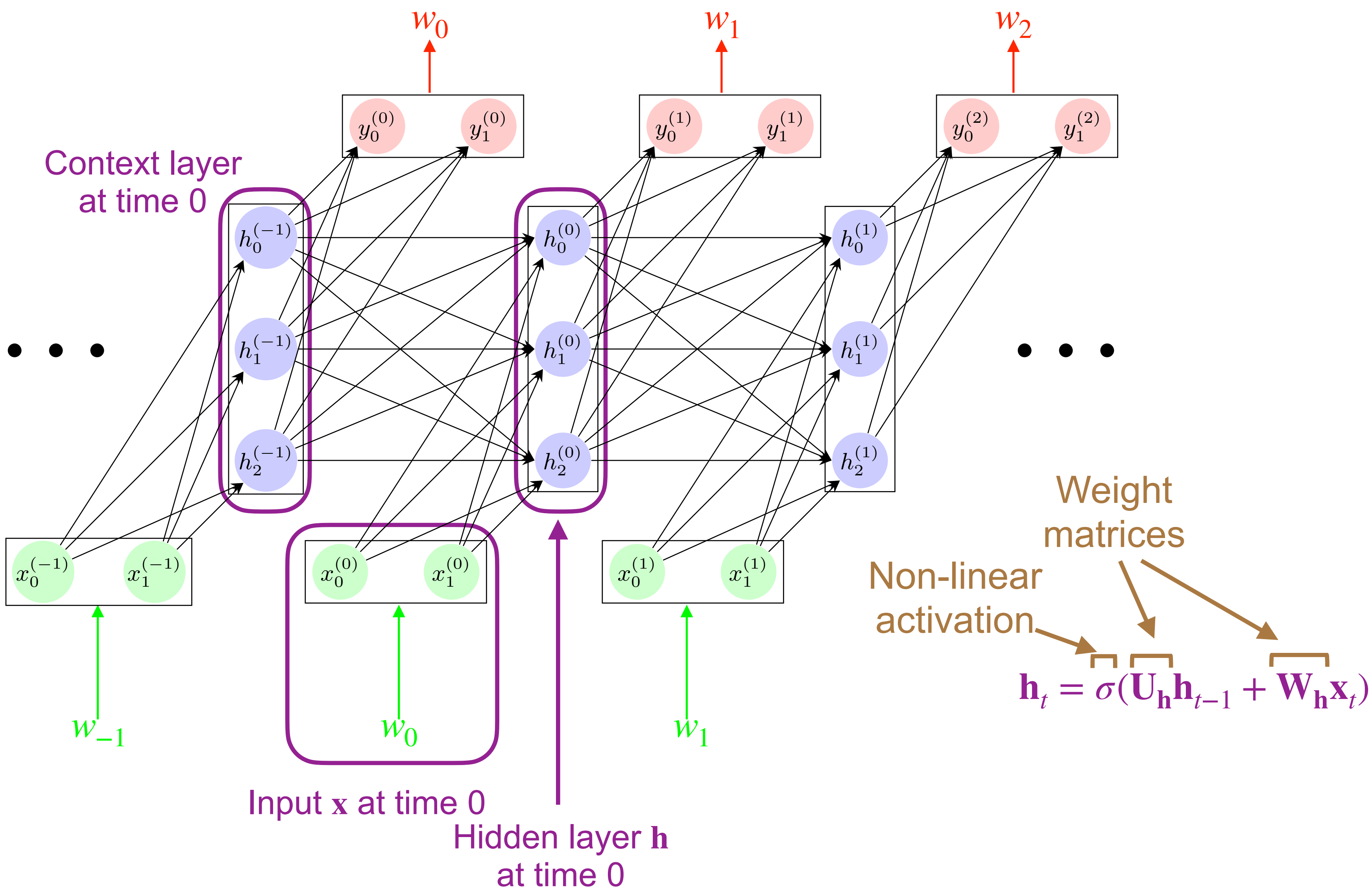

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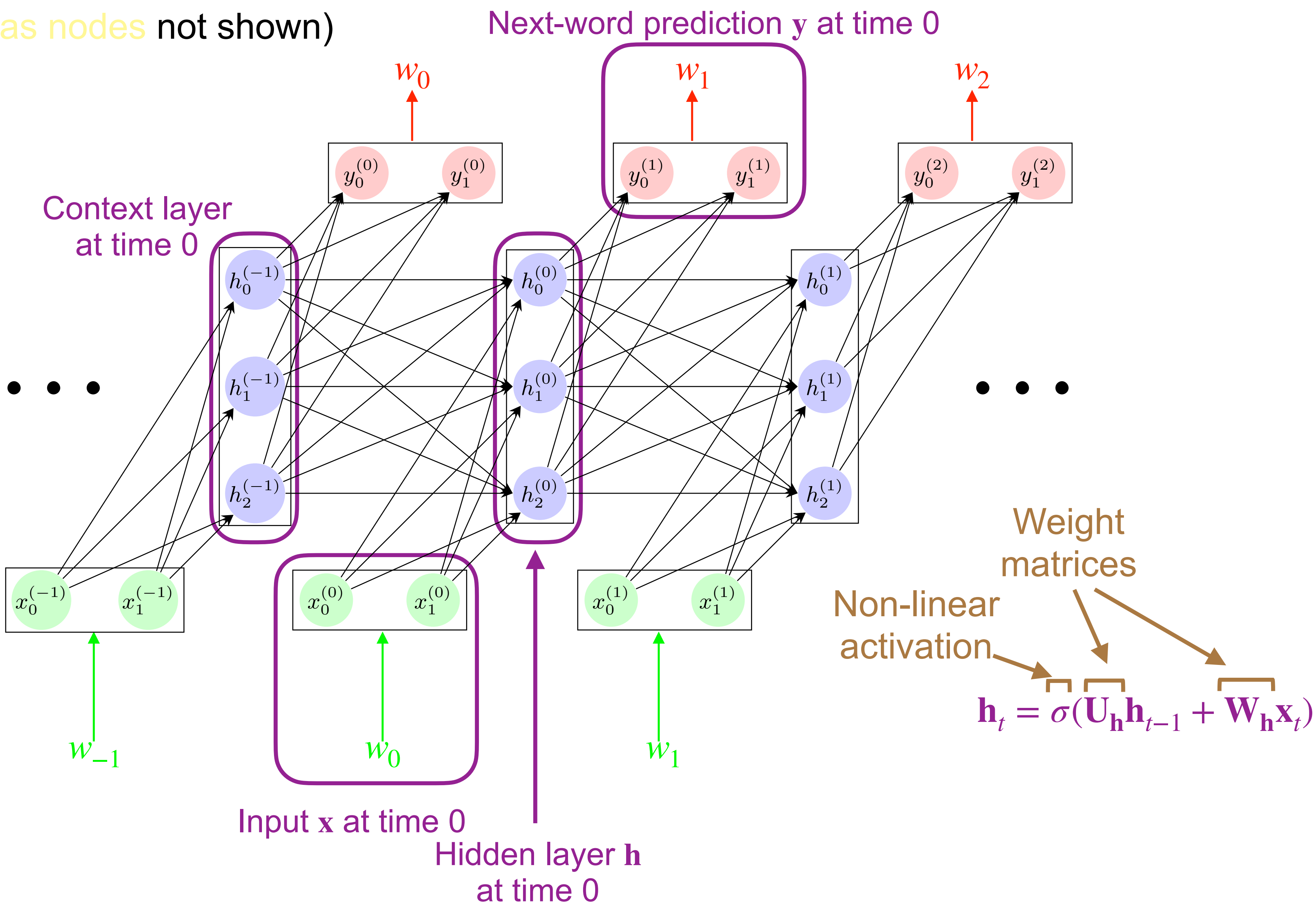

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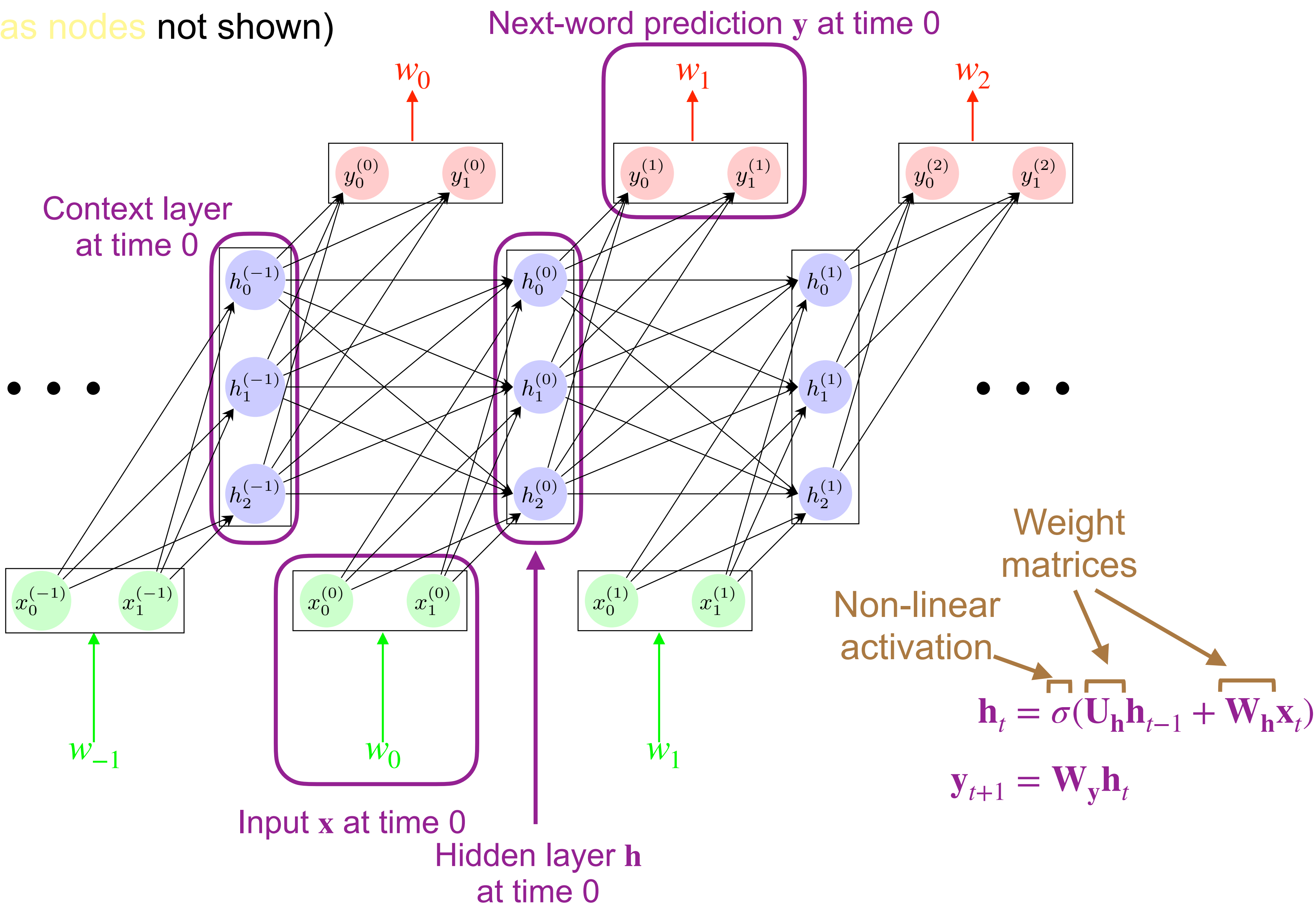

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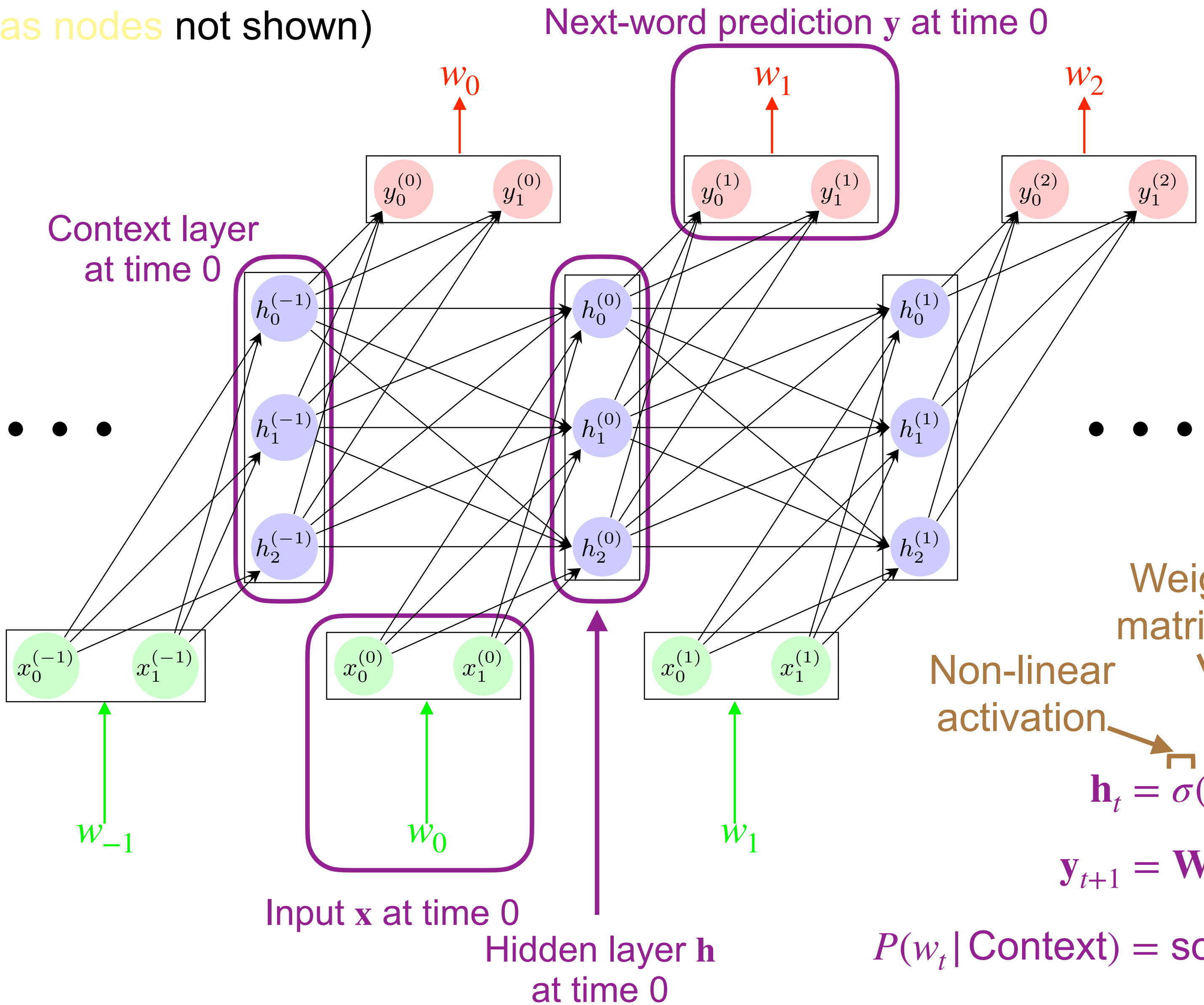


Weight matrices

Non-linear activation

$$\mathbf{h}_t = \sigma(\mathbf{U_h}\mathbf{h}_{t-1} + \mathbf{W_h}\mathbf{x}_t)$$

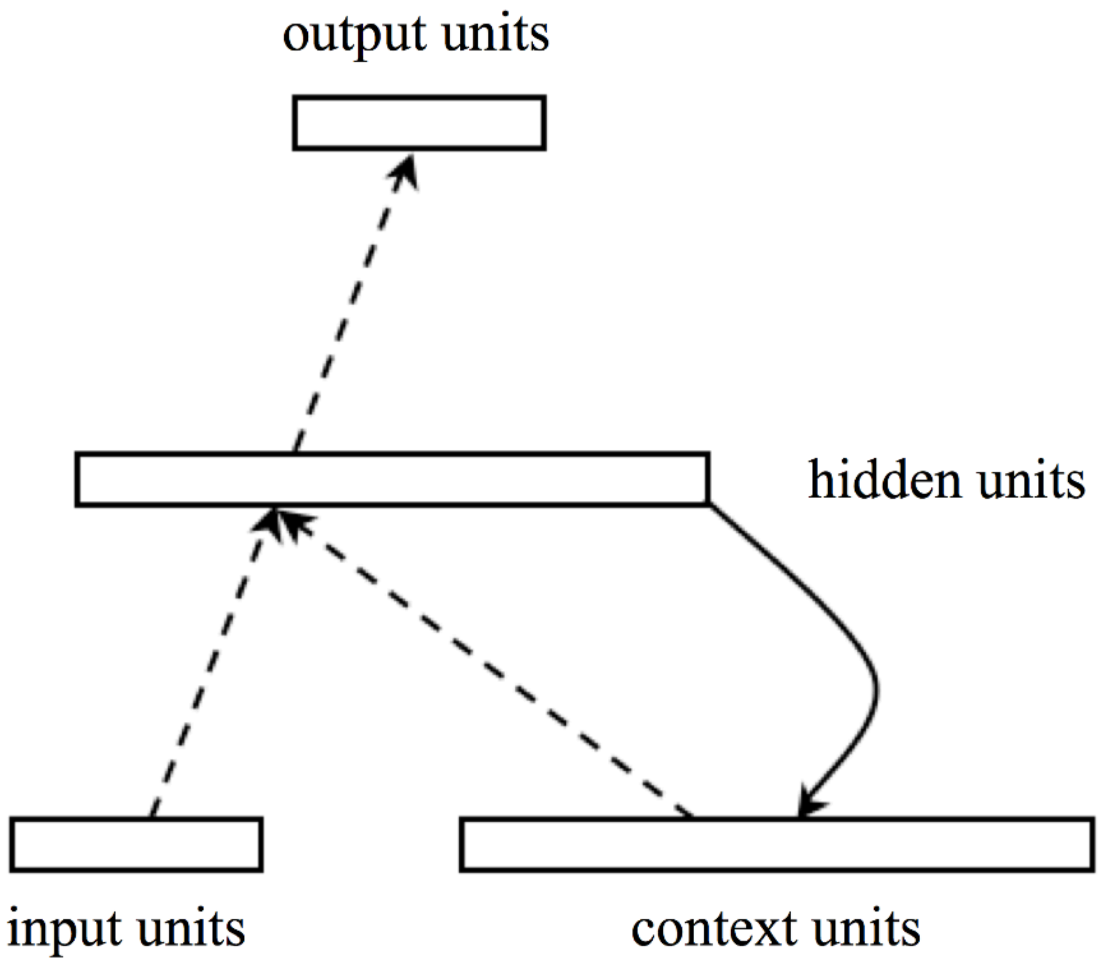
$$\mathbf{y}_{t+1} = \mathbf{W_y}\mathbf{h}_t$$

$$P(w_t \mid \text{Context}) = \text{softmax}(\mathbf{y}_t)$$

(Elman, 1990)

SRN "rolled up" and unrolled

- A “rolled-up” representation (Elman, 1990); and unrolled:



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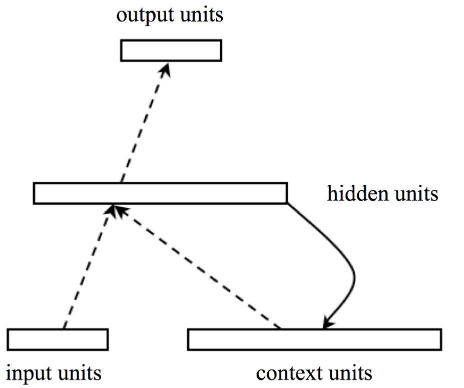

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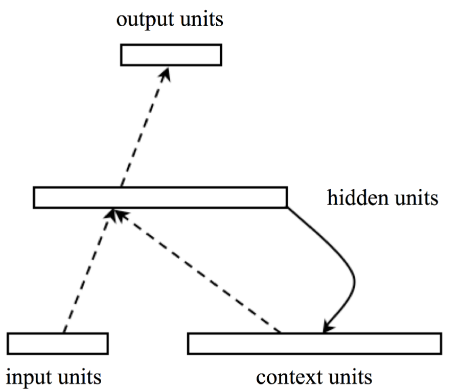

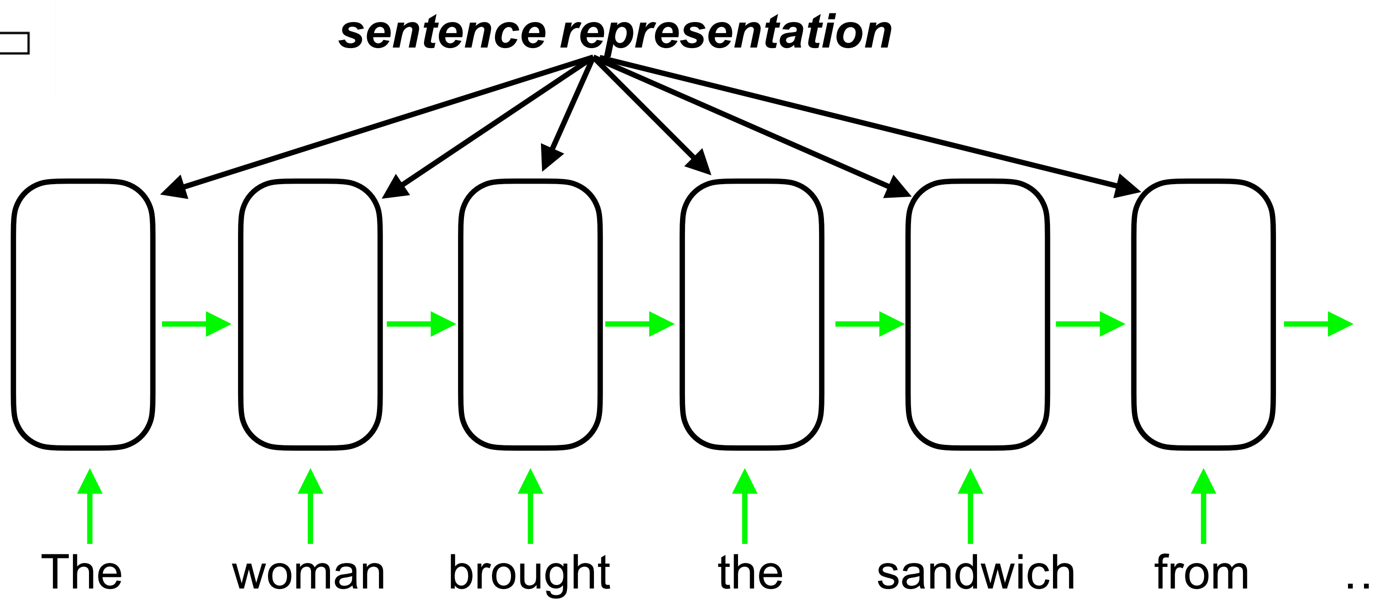

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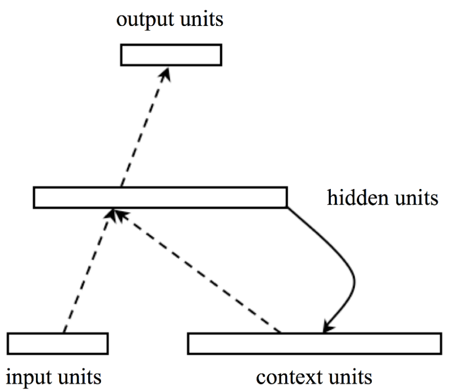

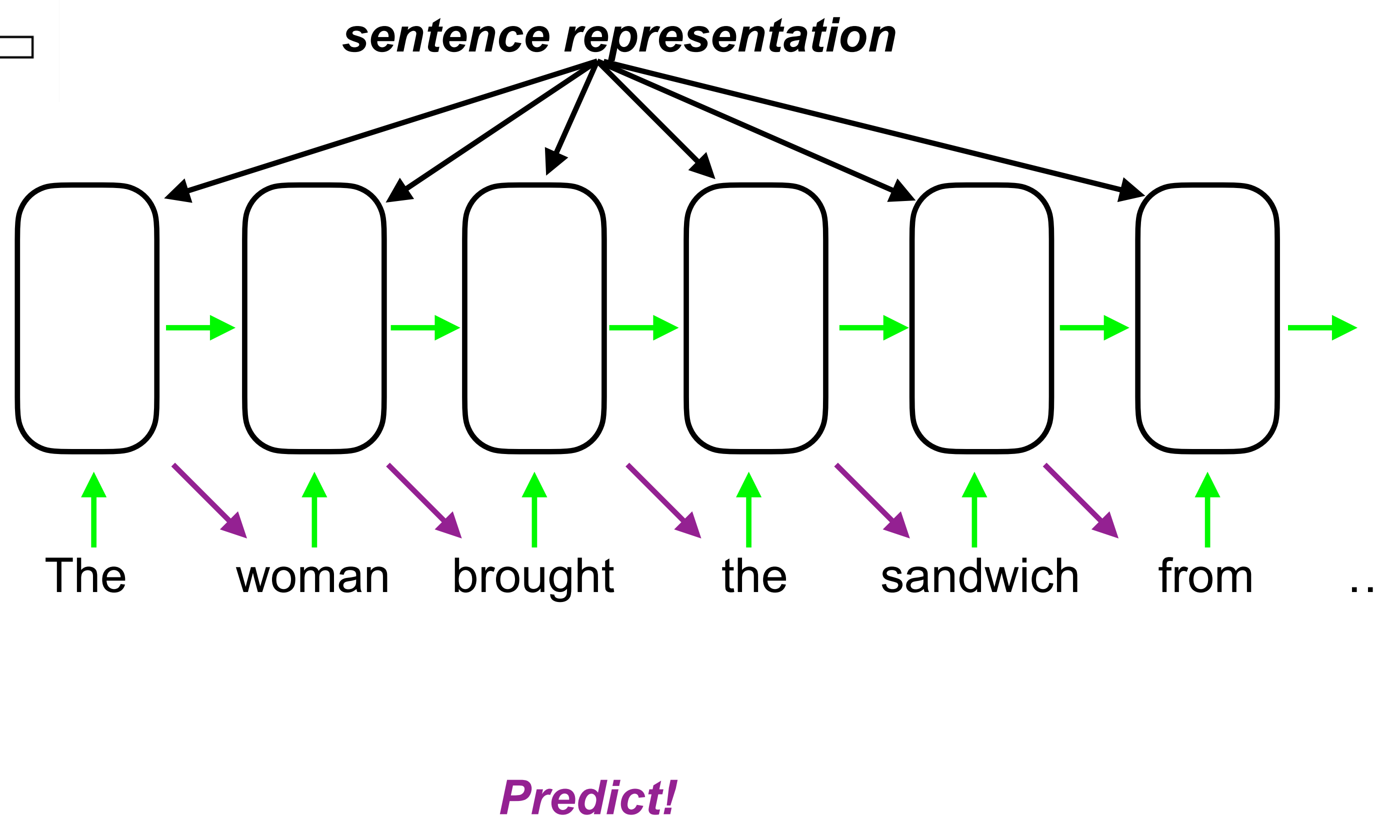

Learning with artificial language input

TABLE 3
Categories of Lexical Items Used in Sentence Simulation

Category	Examples
NOUN-HUM	man, woman
NOUN-ANIM	cat, mouse
NOUN-INANIM	book, rock
NOUN-AGRESS	dragon, monster
NOUN-FRAG	glass, plate
NOUN-FOOD	cookie, break
VERB-INTRAN	think, sleep
VERB-TRAN	see, chase
VERB-AGPAT	move, break
VERB-PERCEPT	smell, see
VERB-DESTROY	break, smash
VERB-EAT	eat

TABLE 4
Templates for Sentence Generator

WORD 1	WORD 2	WORD 3
NOUN-HUM	VERB-EAT	NOUN-FOOD
NOUN-HUM	VERB-PERCEPT	NOUN-INANIM
NOUN-HUM	VERB-DESTROY	NOUN-FRAG
NOUN-HUM	VERB-INTRAN	
NOUN-HUM	VERB-TRAN	NOUN-HUM
NOUN-HUM	VERB-AGPAT	NOUN-INANIM
NOUN-HUM	VERB-AGPAT	
NOUN-ANIM	VERB-EAT	NOUN-FOOD
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NOUN-ANIM	VERB-AGPAT	
NOUN-INANIM	VERB-AGPAT	
NOUN-AGRESS	VERB-DESTROY	NOUN-FRAG
NOUN-AGRESS	VERB-EAT	NOUN-HUM
NOUN-AGRESS	VERB-EAT	NOUN-ANIM
NOUN-AGRESS	VERB-EAT	NOUN-FOOD

(Elman, 1990)

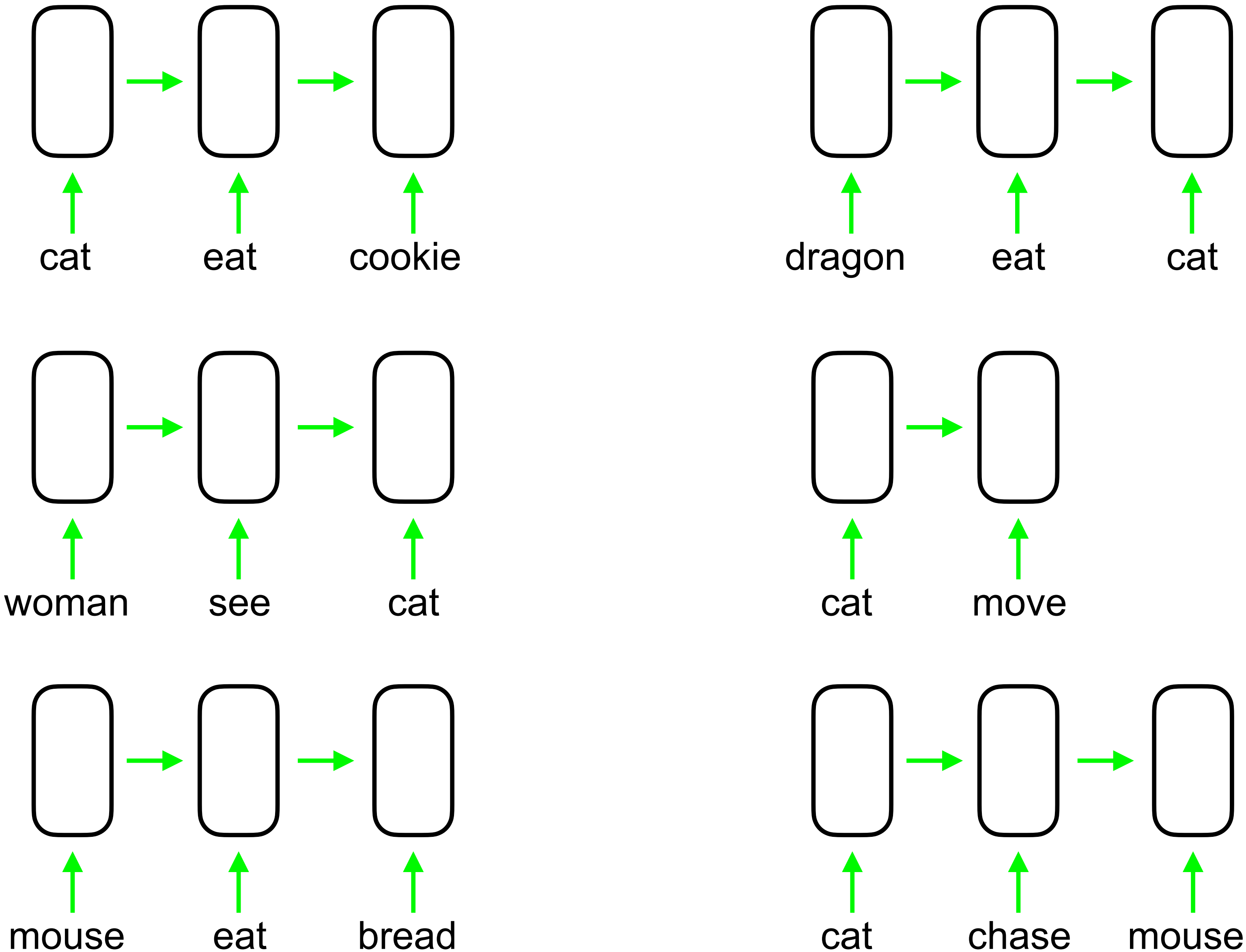
Used *localist* word representations

Fragment of Training Sequences for Sentence Simulation

Input	Output
0000000000000000000000000000010 (*woman*)	0000000000000000000000000010000 (*smash*)
0000000000000000000000000010000 (*smash*)	0000000000000000000001000000000 (*plate*)
0000000000000000000001000000000 (*plate*)	0000010000000000000000000000000 (*cat*)
0000010000000000000000000000000 (*cat*)	0000000000000000000100000000000 (*move*)
0000000000000000000100000000000 (*move*)	0000000000000000100000000000000 (*man*)
0000000000000000100000000000000 (*man*)	0001000000000000000000000000000 (*break*)
0001000000000000000000000000000 (*break*)	0000100000000000000000000000000 (*car*)
0000100000000000000000000000000 (*car*)	0100000000000000000000000000000 (*boy*)
0100000000000000000000000000000 (*boy*)	0000000000000000000100000000000 (*move*)
0000000000000000000100000000000 (*move*)	0000000000001000000000000000000 (*girl*)
0000000000001000000000000000000 (*girl*)	0000000000100000000000000000000 (*eat*)
0000000000100000000000000000000 (*eat*)	0010000000000000000000000000000 (*bread*)
0010000000000000000000000000000 (*bread*)	0000000010000000000000000000000 (*dog*)
0000000010000000000000000000000 (*dog*)	0000000000000000000100000000000 (*move*)
0000000000000000000100000000000 (*move*)	0000000000000000001000000000000 (*mouse*)
0000000000000000001000000000000 (*mouse*)	0000000000000000001000000000000 (*mouse*)
0000000000000000001000000000000 (*mouse*)	0000000000000000000100000000000 (*move*)
0000000000000000000100000000000 (*move*)	1000000000000000000000000000000 (*book*)
1000000000000000000000000000000 (*book*)	0000000000000001000000000000000 (*lion*)

(Elman, 1990)

Learning word classes



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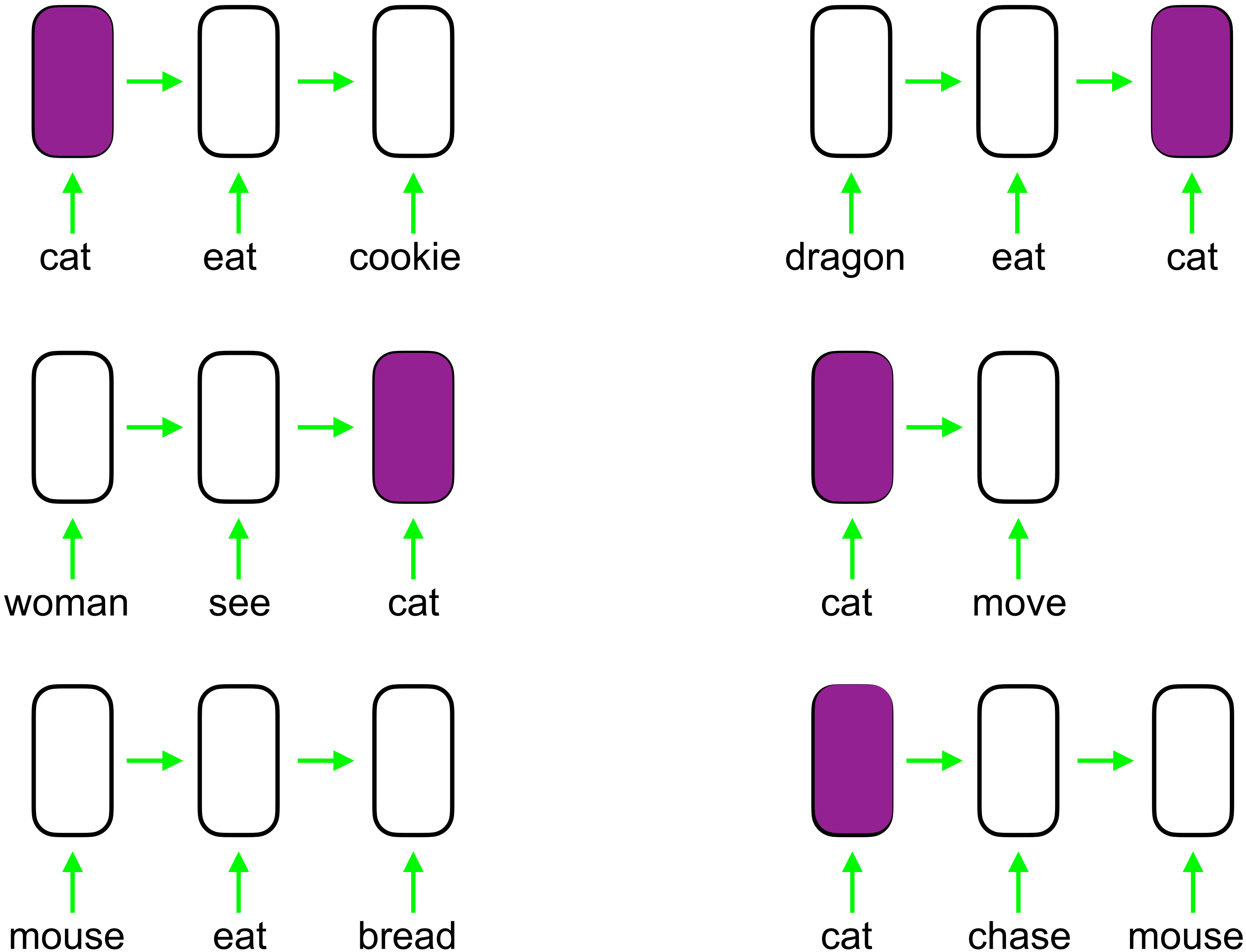

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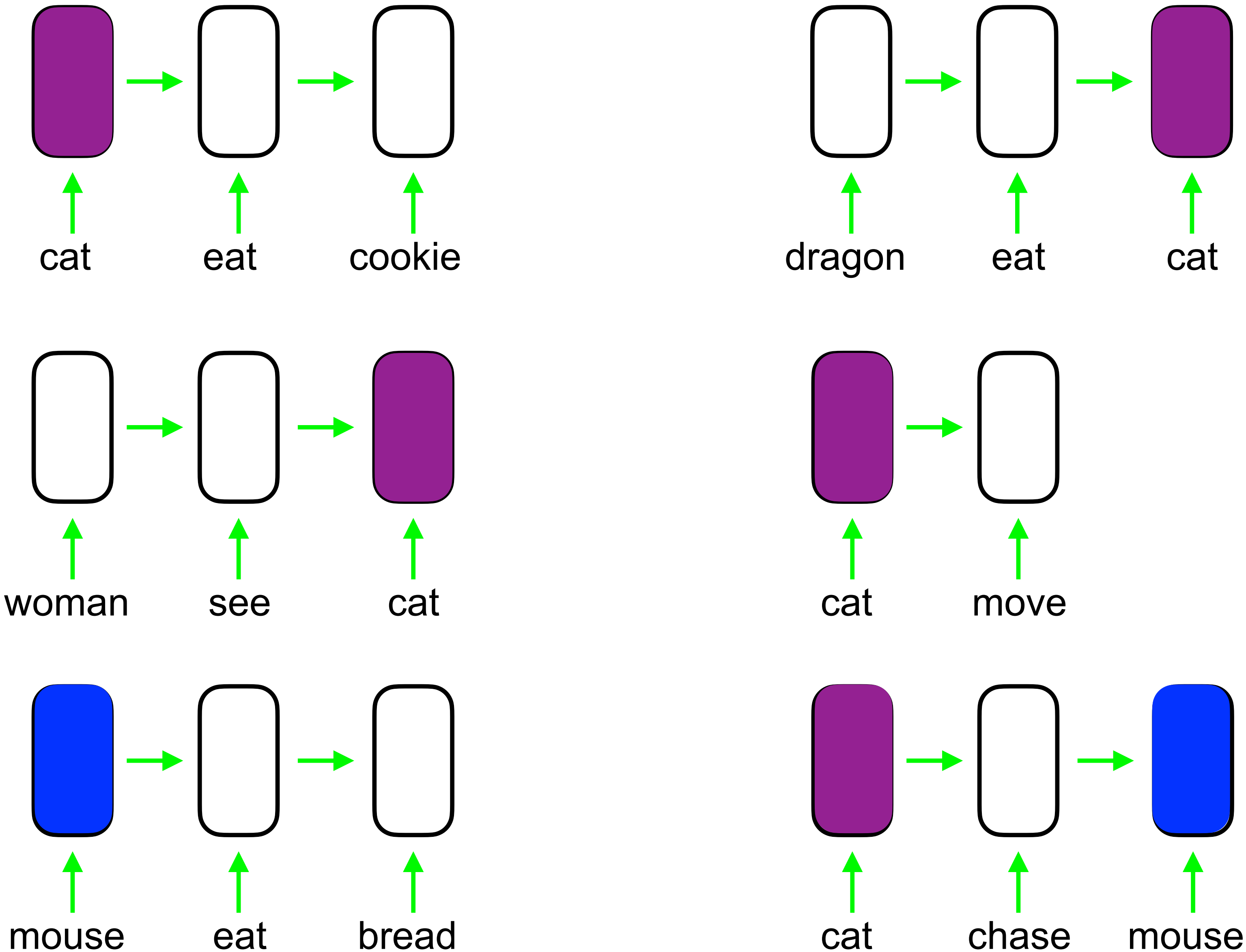
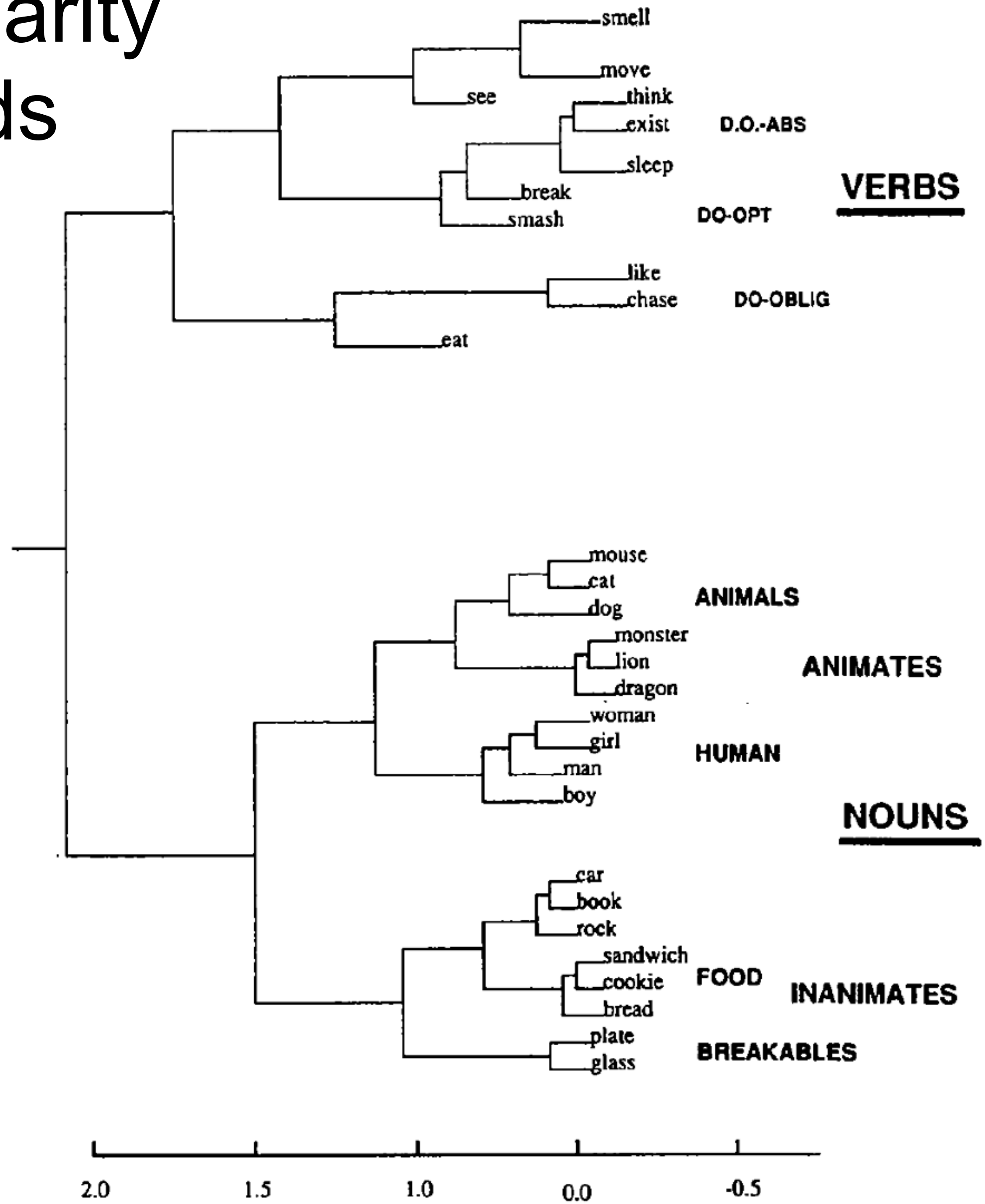


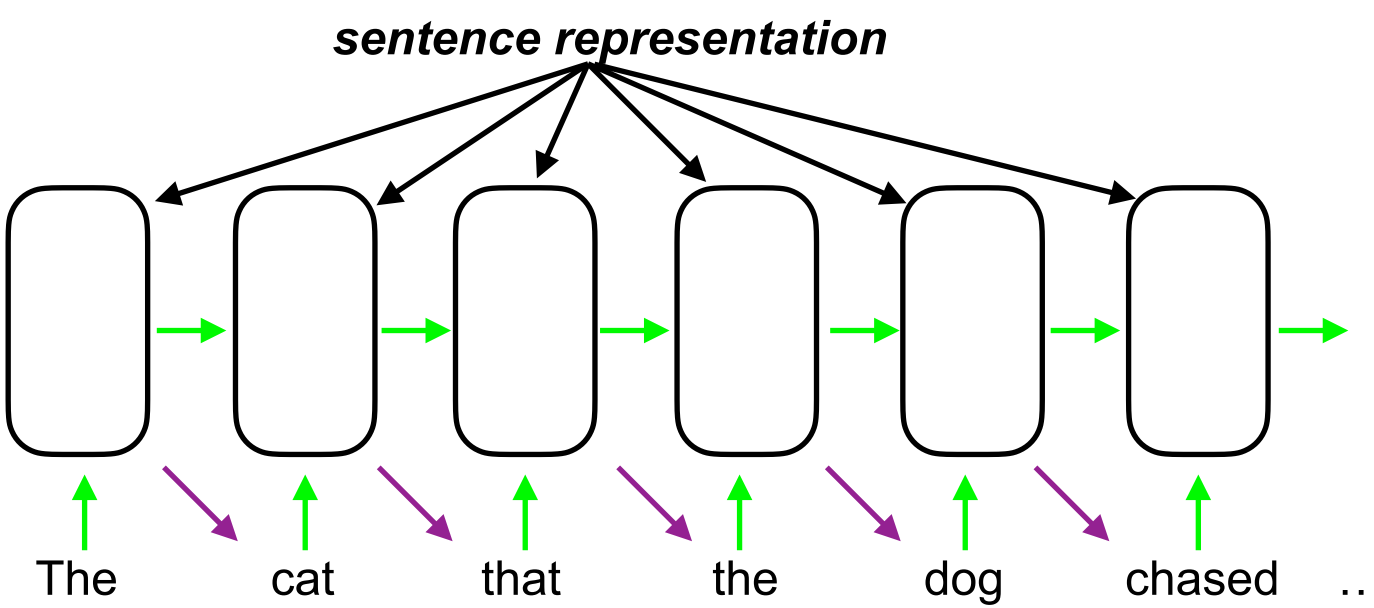
Discovered similarity structure of words



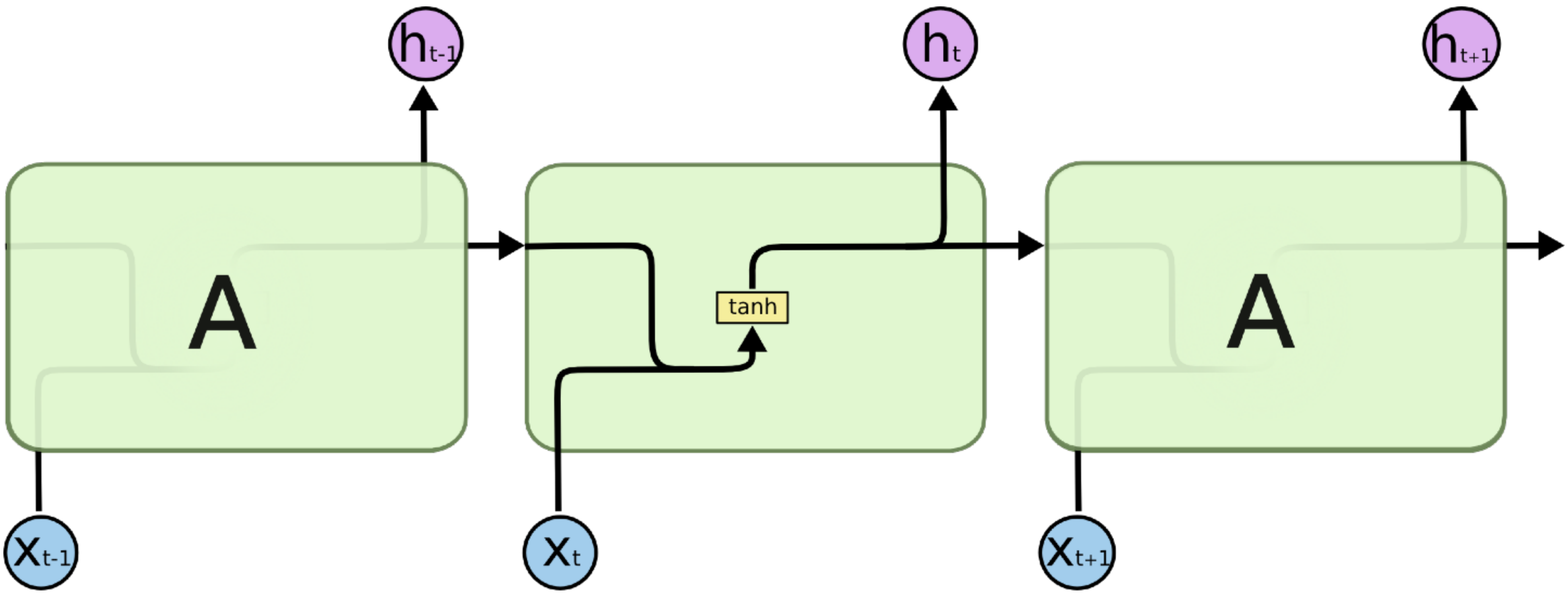
(Elman, 1990)

Beyond the simple recurrent network

- The SRN has a very strong ***linear locality bias***
- But natural language syntax is characterized by ***hierarchical structure***
- SRNs can learn hierarchy (Elman, 1991), but ***it is hard***—their inductive bias disfavors it



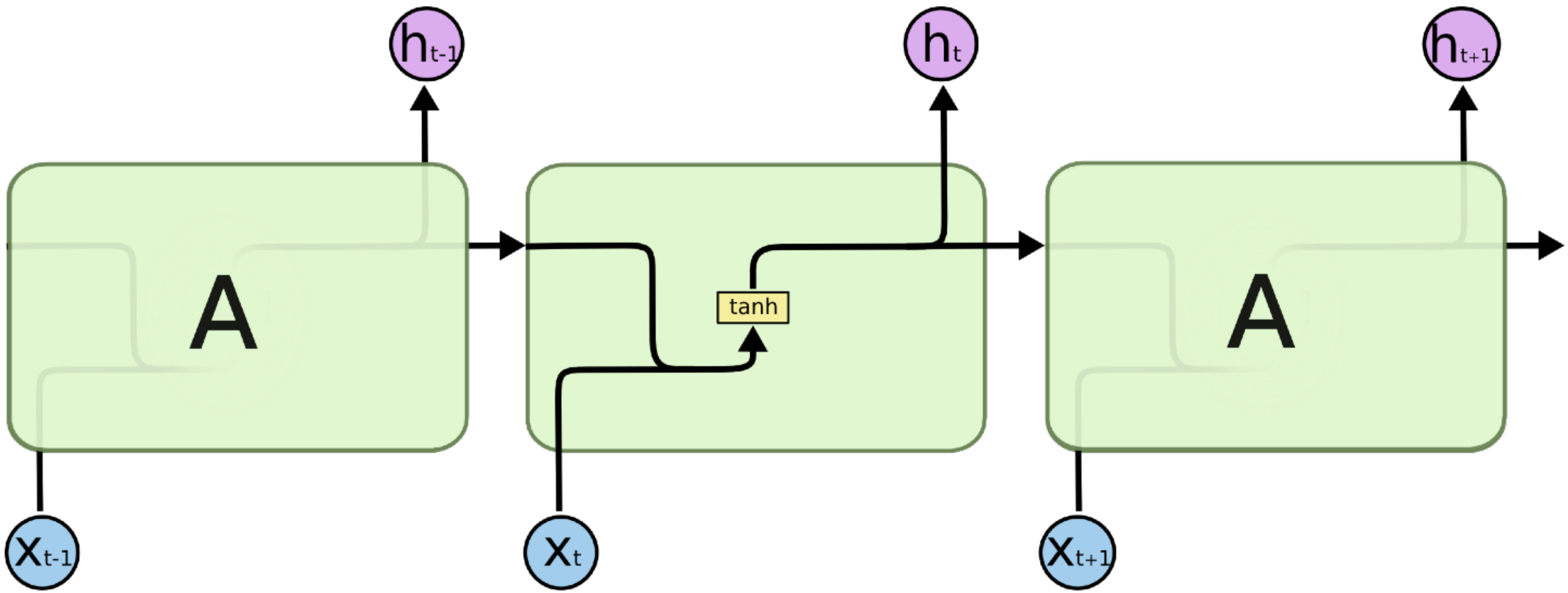
More sophisticated recurrent units



visualization due to Christopher Olah, http://colah.github.io/posts/2015-08-Understanding-LSTMs/

More sophisticated recurrent units

- Another view of an unrolled SRN:



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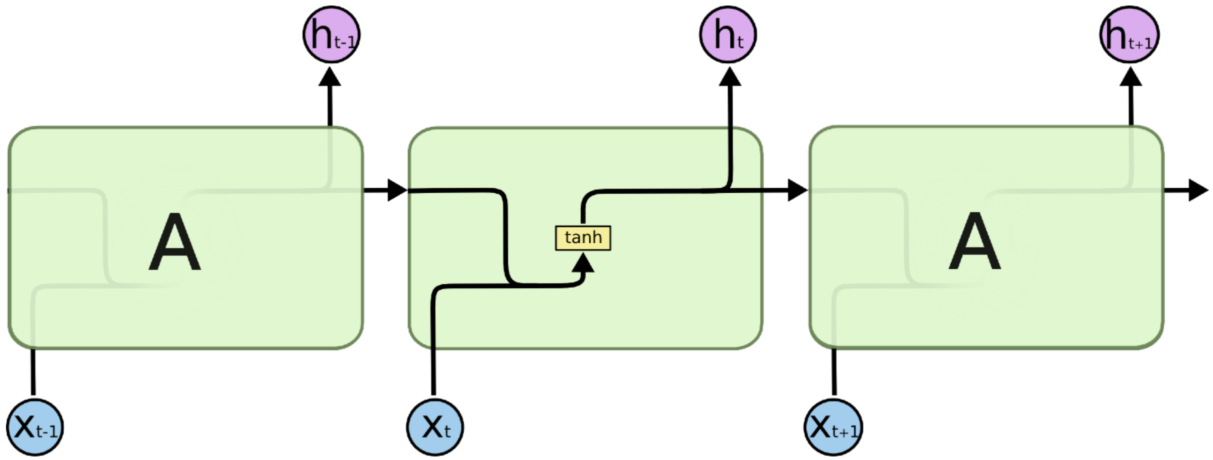

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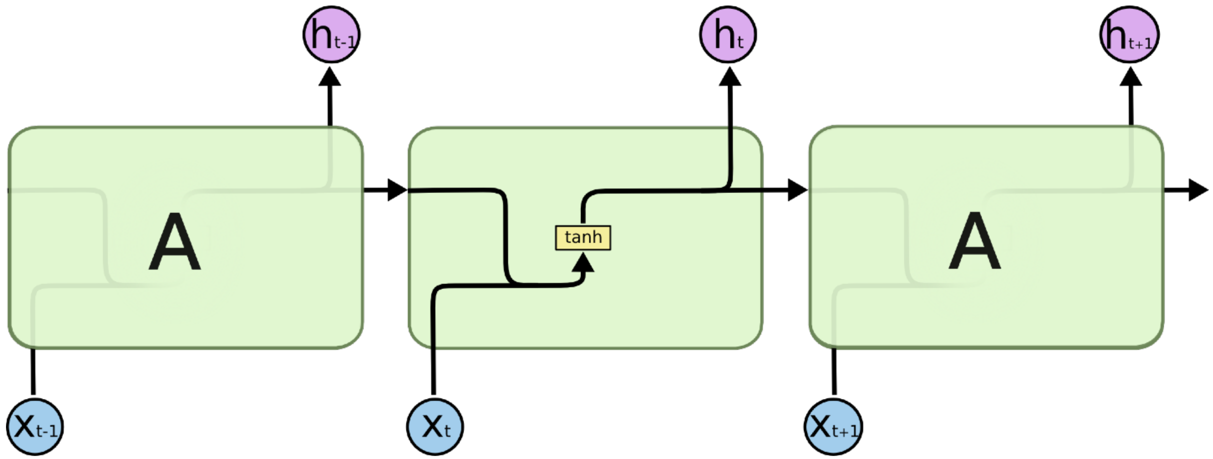
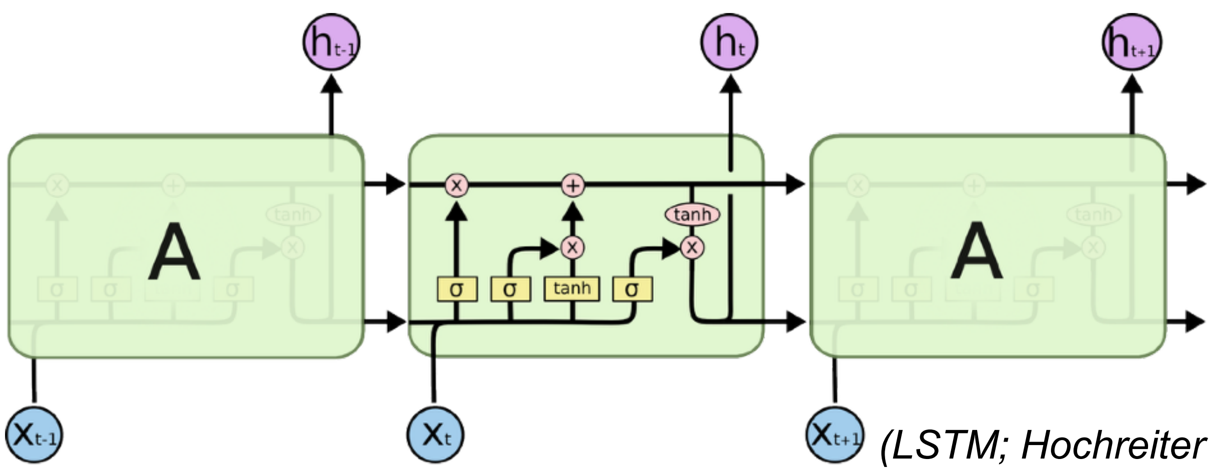


- Keep the recurrent structure and "swap in" a new unit:



(LSTM; Hochreiter & Schmidhuber, 1997)

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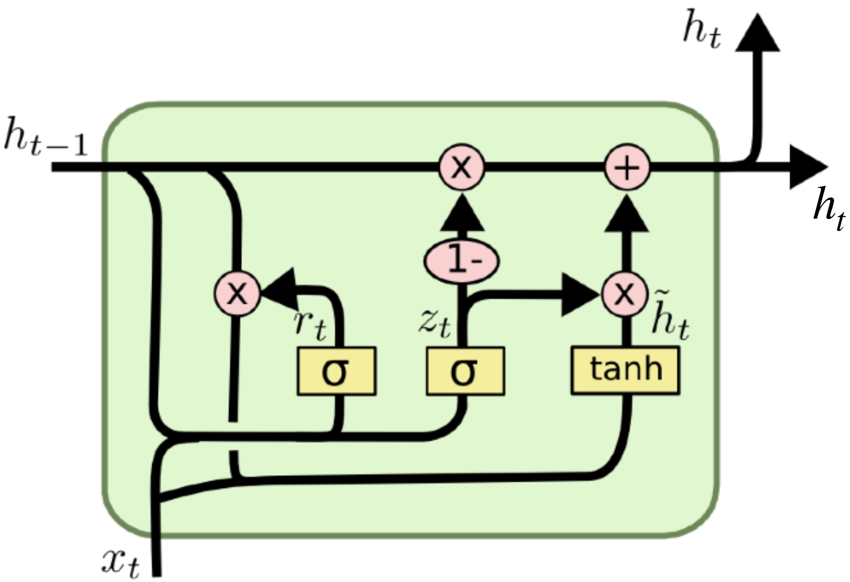
Gated Recurrent Unit (GRU) architecture



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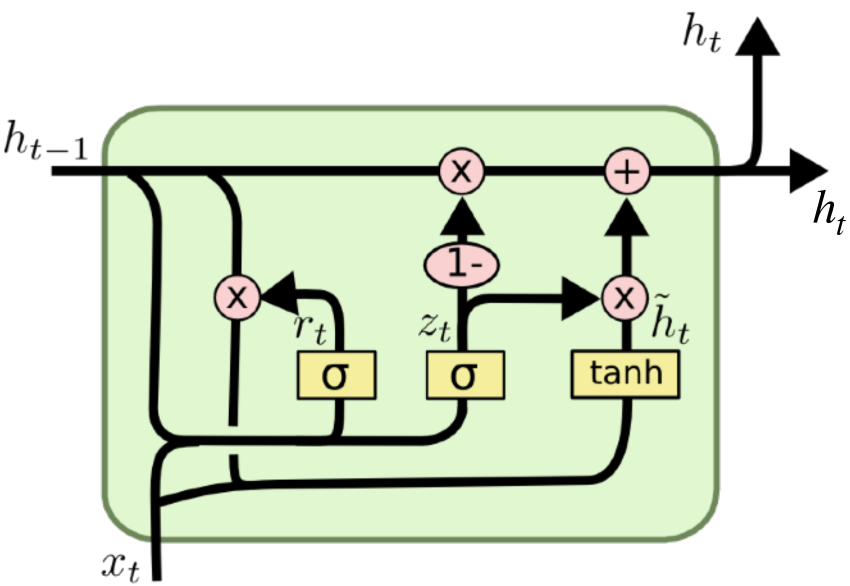


$$\mathbf{r}_t = \sigma(\mathbf{W}_r \mathbf{x}_t + \mathbf{U}_r \mathbf{h}_{t-1})$$

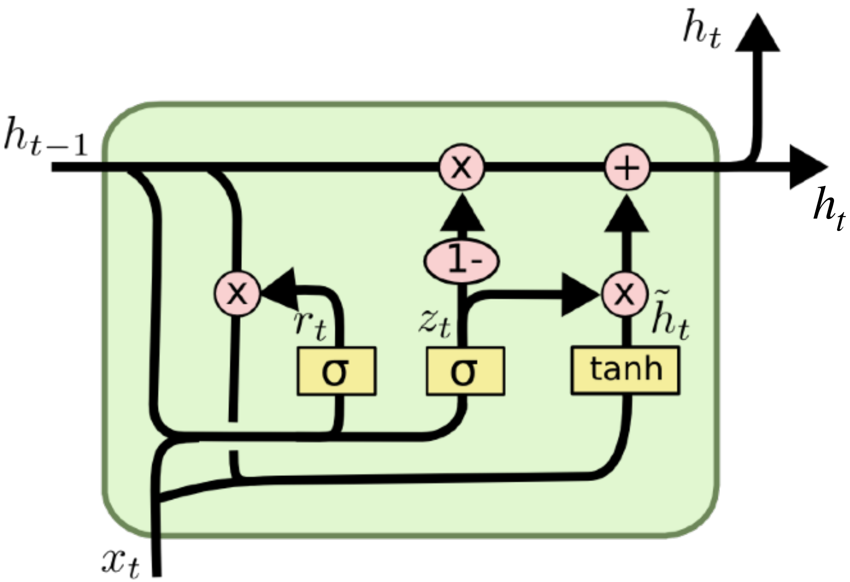
$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{W}\mathbf{x}_t + \mathbf{U}(\mathbf{r}_t \odot \mathbf{h}_{t-1}))$$

$$\mathbf{z}_t = \sigma(\mathbf{W}_z \mathbf{x}_t + \mathbf{U}_z \mathbf{h}_{t-1})$$

$$\mathbf{h}_t = (1 - \mathbf{z}_t) \odot \mathbf{h}_{t-1} + \mathbf{z}_t \odot \tilde{\mathbf{h}}_t$$

visualization due to Christopher Olah, http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Gated Recurrent Unit (GRU) architecture



logistic/sigmoid activation function

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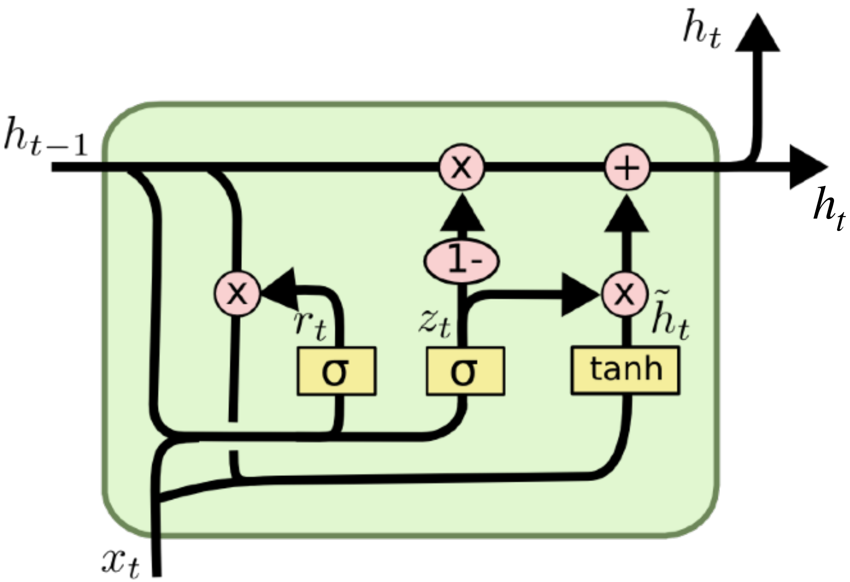

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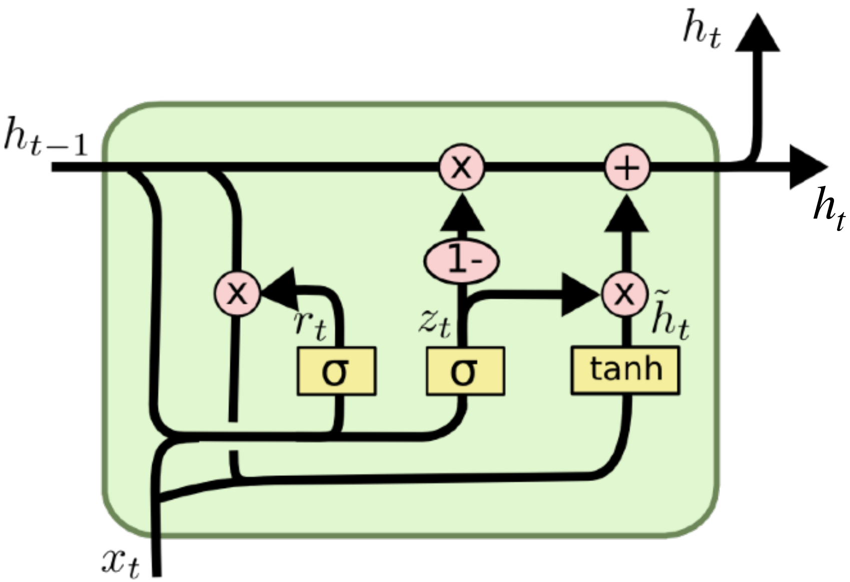

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(e.g., $\langle 1,2,3 \rangle \odot \langle 0.5,2,1 \rangle = \langle 0.5,4,3 \rangle$)

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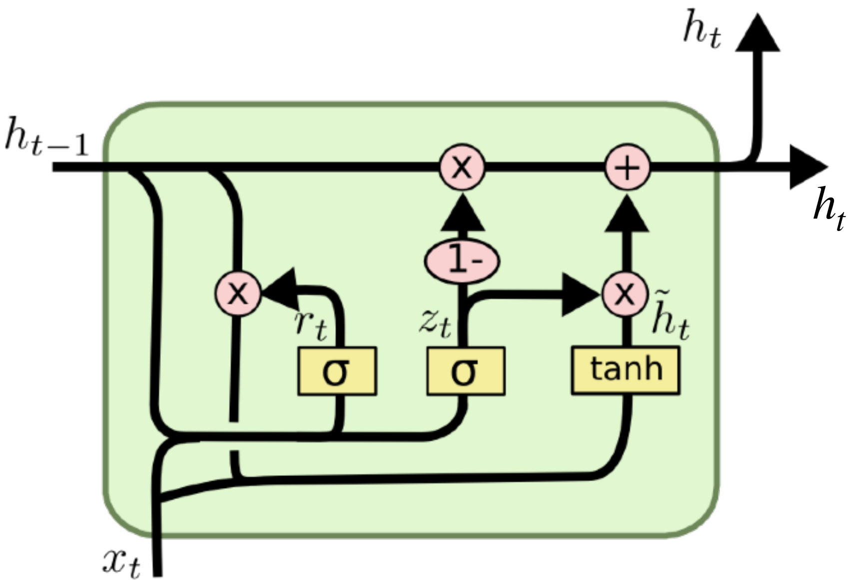

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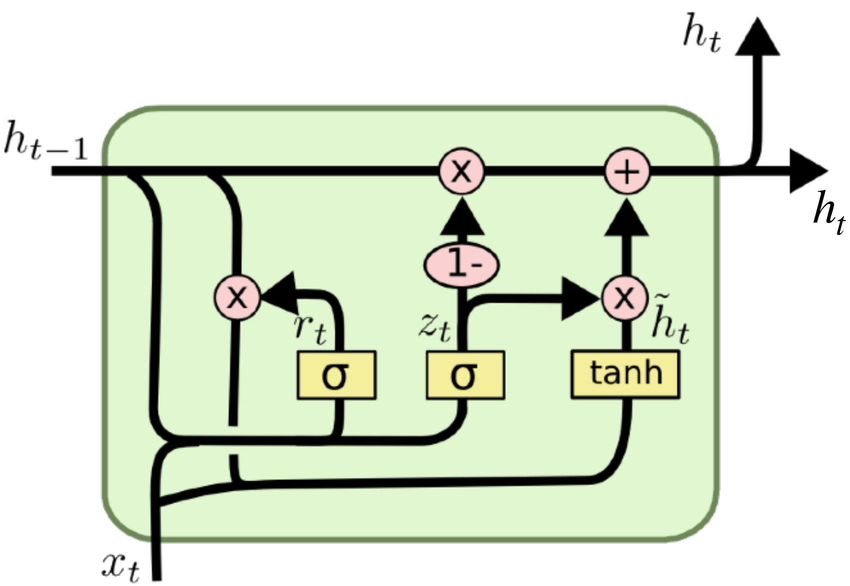

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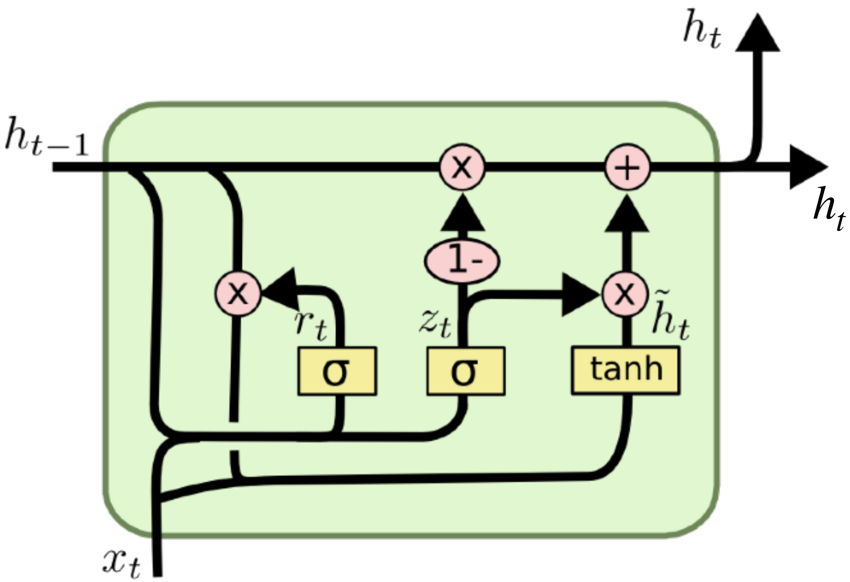

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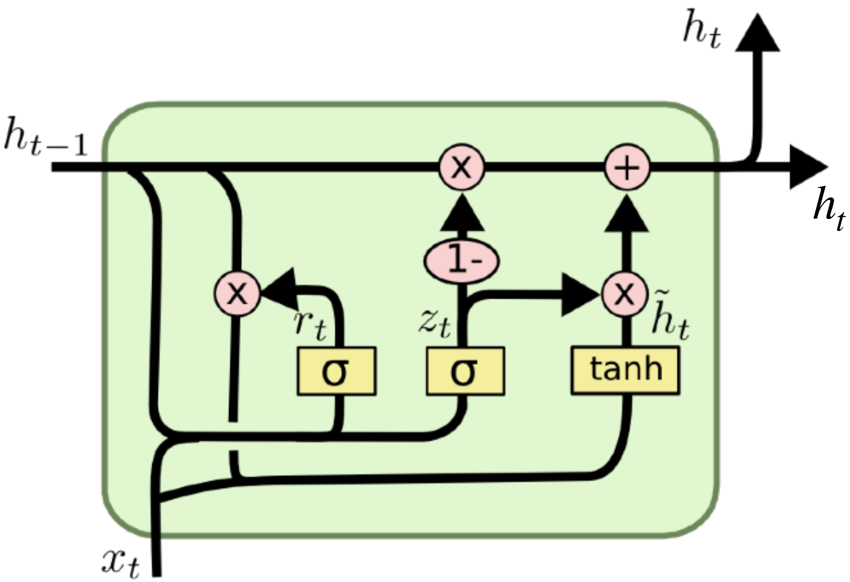

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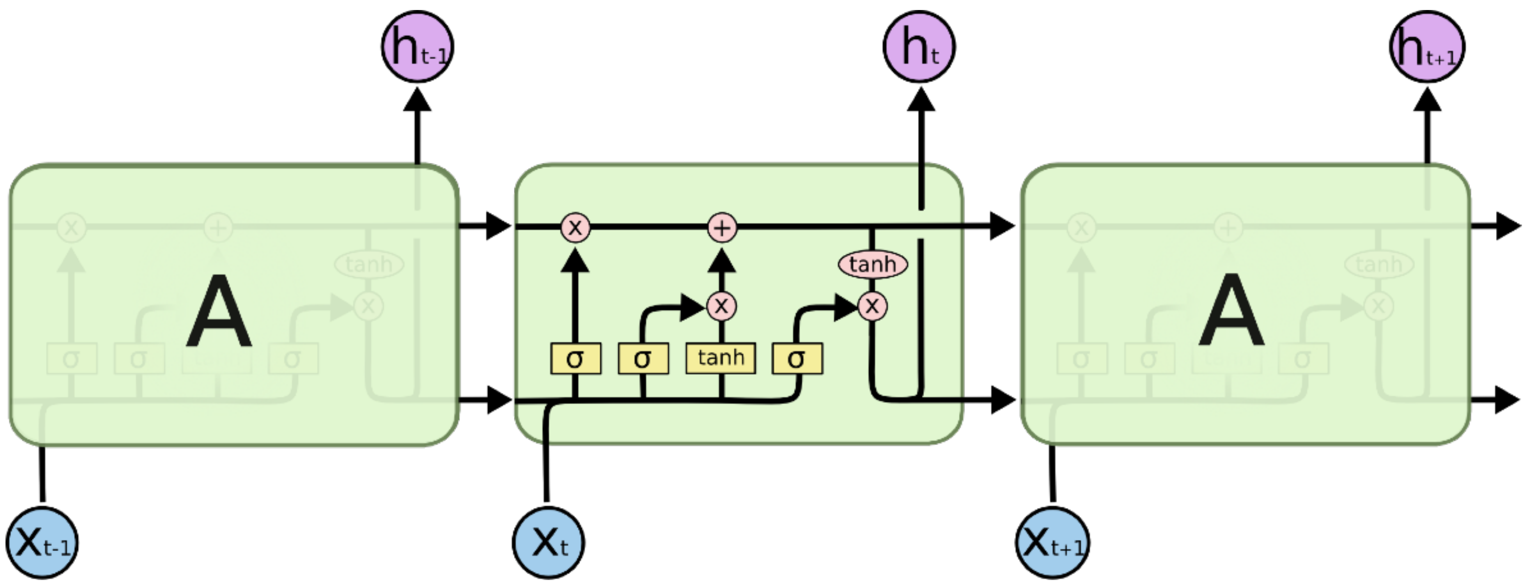
visualization due to Christopher Olah, http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Long short-term memory (LSTM) units

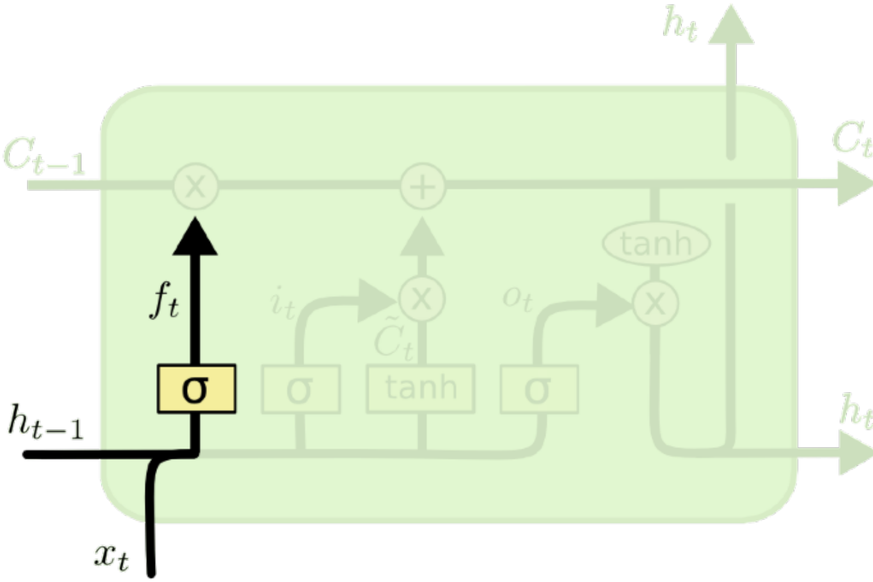


(Hochreiter & Schmidhuber, 1997)

visualization due to Christopher Olah, http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Inside the LSTM unit

- The "hidden layer" $\mathbf{h}_{t-1}$ was used to predict element *t* of the sequence
- It now gets passed through a "forget gate"



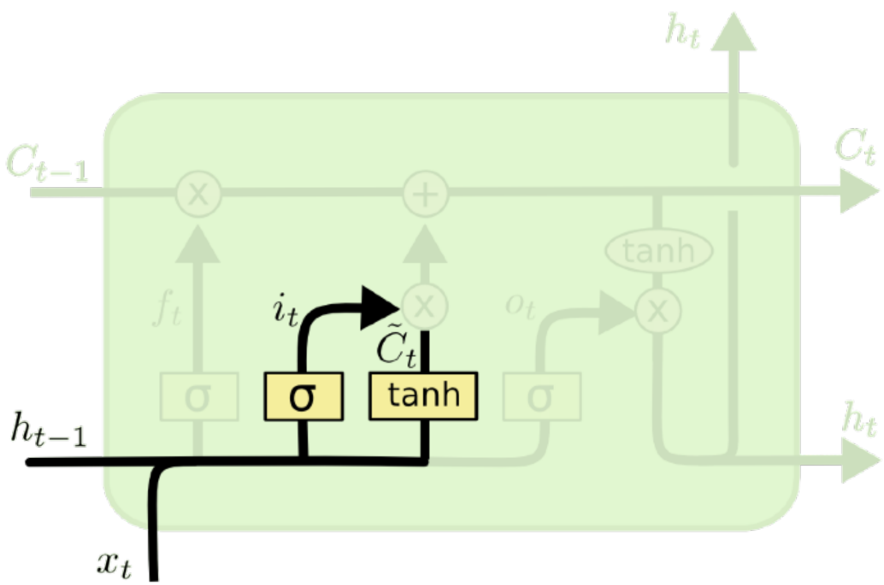
$$\mathbf{f}_t = \sigma(\mathbf{U_f}\mathbf{h}_{t-1} + \mathbf{W}_f\mathbf{x}_t)$$

(Hochreiter & Schmidhuber, 1997)

visualization due to Christopher Olah, http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Inside the LSTM unit

- Other information from h_{t-1} gets put into the memory store



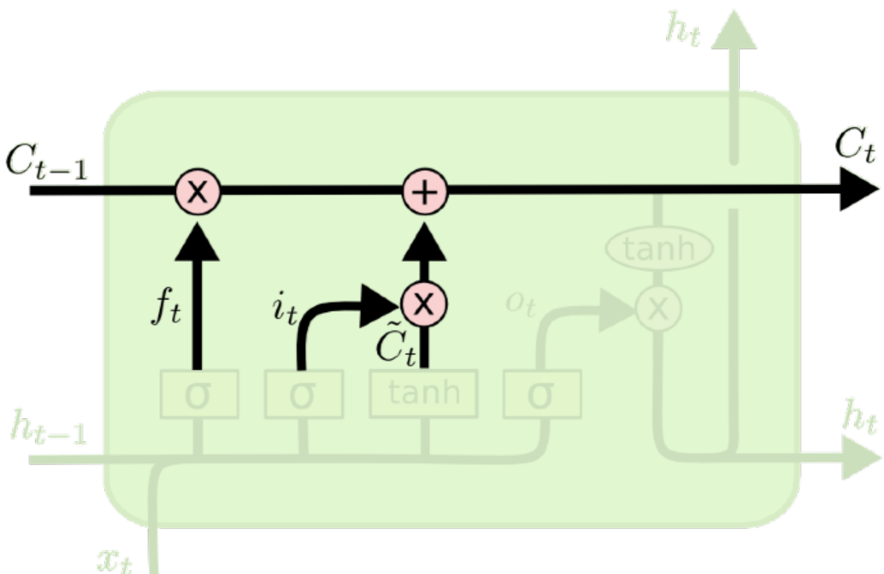
$$\mathbf{i}_t = \sigma(\mathbf{U_i h}_{t-1} + \mathbf{W_i x}_t)$$

$$\tilde{\mathbf{C}}_t = \tanh(\mathbf{U_C h}_{t-1} + \mathbf{W_C x}_t)$$

visualization due to Christopher Olah, http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Inside the LSTM unit

- That information gets integrated into the memory store (which also gets passed on to the future

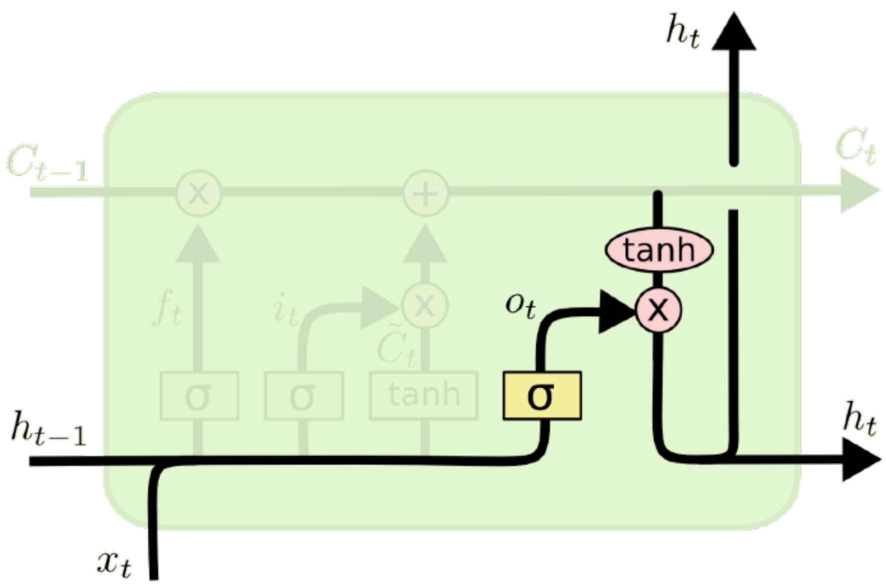


$$\mathbf{C}_t = \mathbf{f}_t \odot \mathbf{C}_{t-1} + \mathbf{i}_t \odot \tilde{\mathbf{C}}_t$$

visualization due to Christopher Olah, http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Inside the LSTM unit

- Finally, we determine the new hidden layer to predict input *t+1*

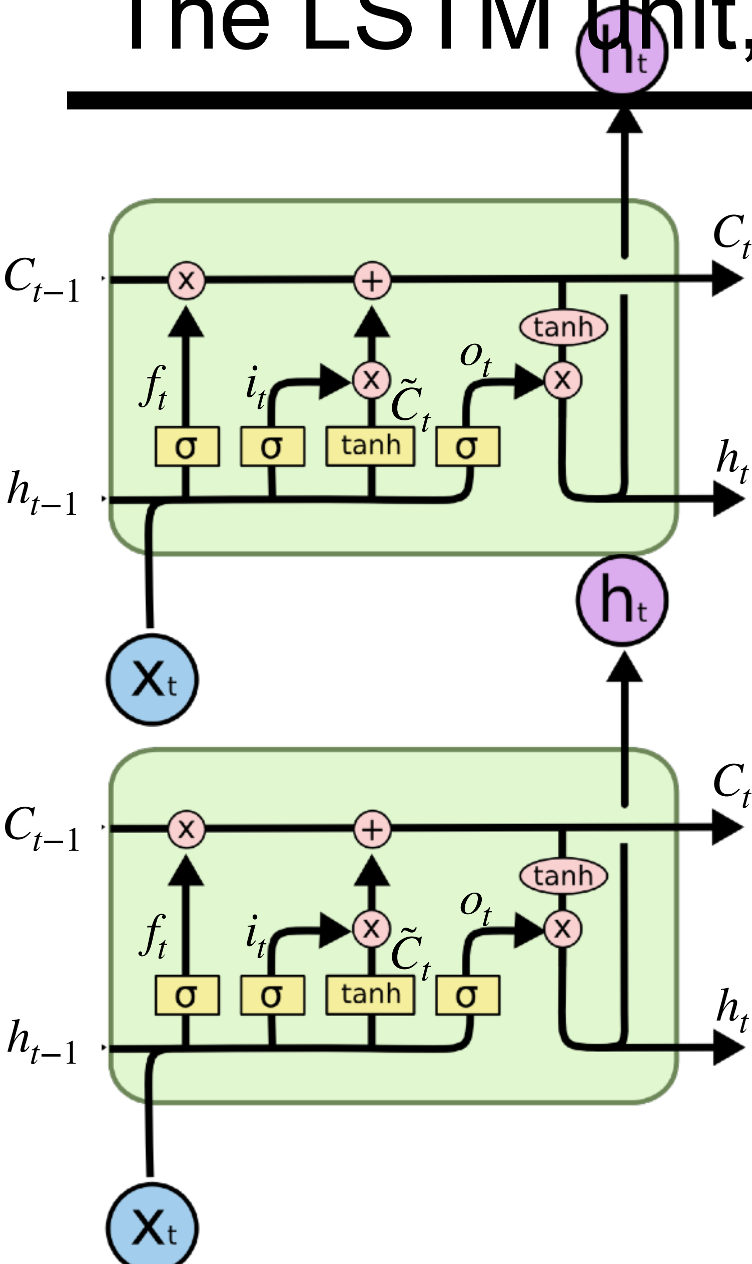


$$\mathbf{o}_t = \sigma(\mathbf{U_o h}_{t-1} + \mathbf{W_o x}_t)$$

$$\mathbf{h}_t = \mathbf{o}_t \odot \tanh(\mathbf{C}_t)$$

visualization due to Christopher Olah, http://colah.github.io/posts/2015-08-Understanding-LSTMs/

The LSTM unit, complete



$$\mathbf{f}_t = \sigma(\mathbf{U_f}\mathbf{h}_{t-1} + \mathbf{W}_f\mathbf{x}_t)$$

$$\mathbf{i}_t = \sigma(\mathbf{U_i}\mathbf{h}_{t-1} + \mathbf{W_i}\mathbf{x}_t)$$

$$\tilde{\mathbf{C}}_t = \tanh(\mathbf{U_C}\mathbf{h}_{t-1} + \mathbf{W_C}\mathbf{x}_t)$$

$$\mathbf{C}_t = \mathbf{f}_t \odot \mathbf{C}_{t-1} + \mathbf{i}_t \odot \tilde{\mathbf{C}}_t$$

$$\mathbf{o}_t = \sigma(\mathbf{U_o}\mathbf{h}_{t-1} + \mathbf{W_o}\mathbf{x}_t)$$

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Learning the classic counting language

$a^n b^n$

(Weiss et al. 2018, ACL)

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Easily generable with a context-free grammar:

S → a b
S → a S b

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```
^ab$
^aabb$
^aaabbb$
^aaaabbbb$
^aaaaabbbbb$
^aaaaaabbbbbb$
^aaaaaaabbbbbbb$
   ⋮
^aaaaaaaaaaaaaaaaaaaabbbbbbbbbbbbbbbbbbbb$
```

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D

D_{train}

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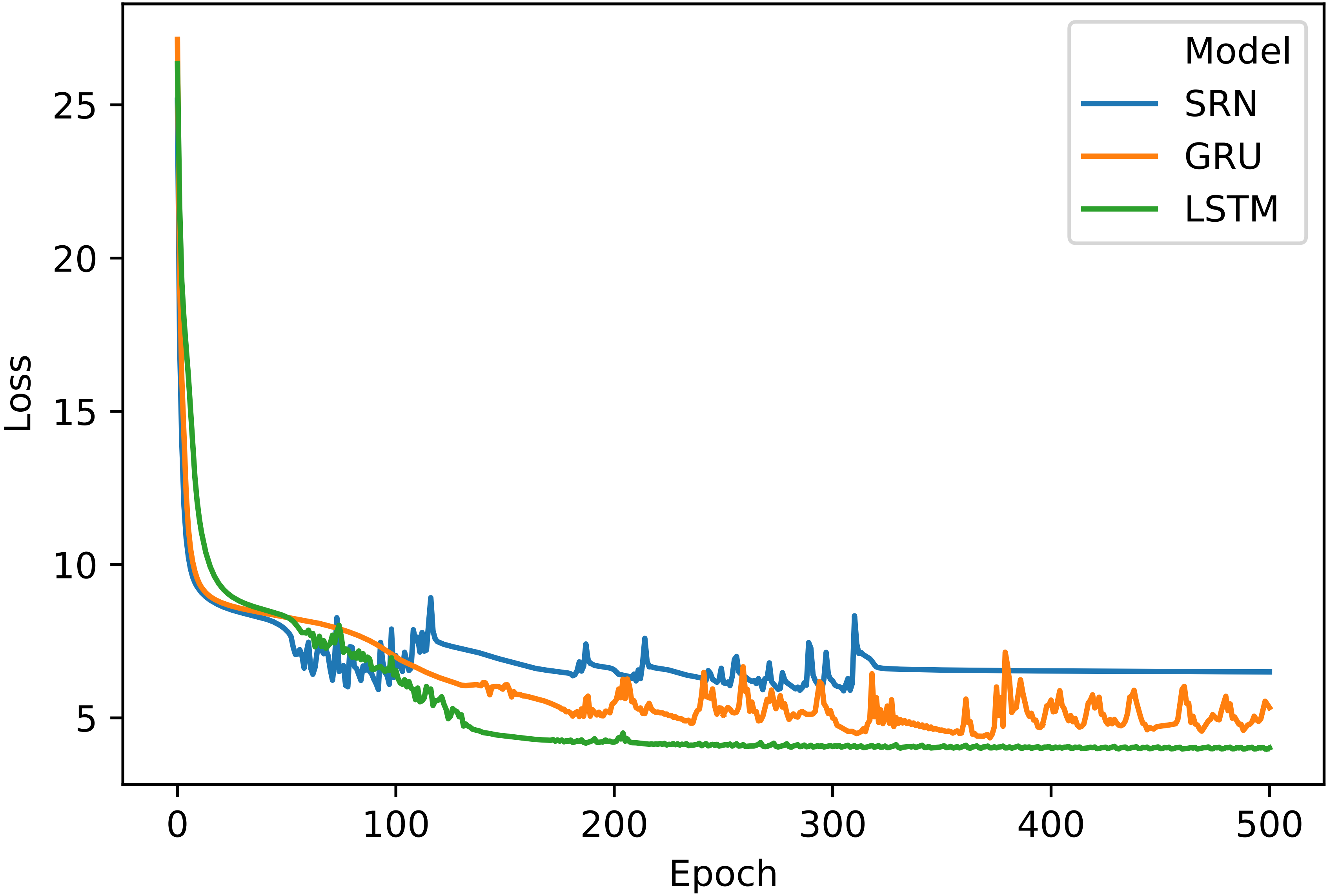
N=20 N=20

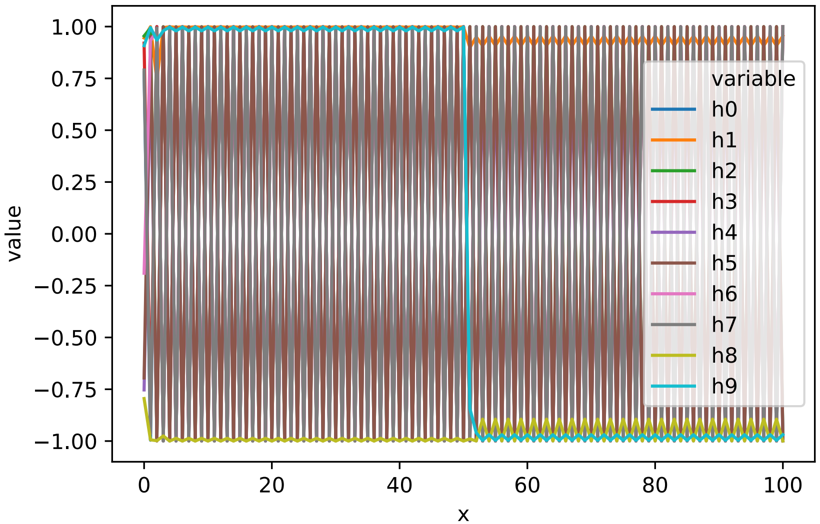
(Weiss et al. 2018, ACL)

Training recurrent architectures on $a^n b^n$

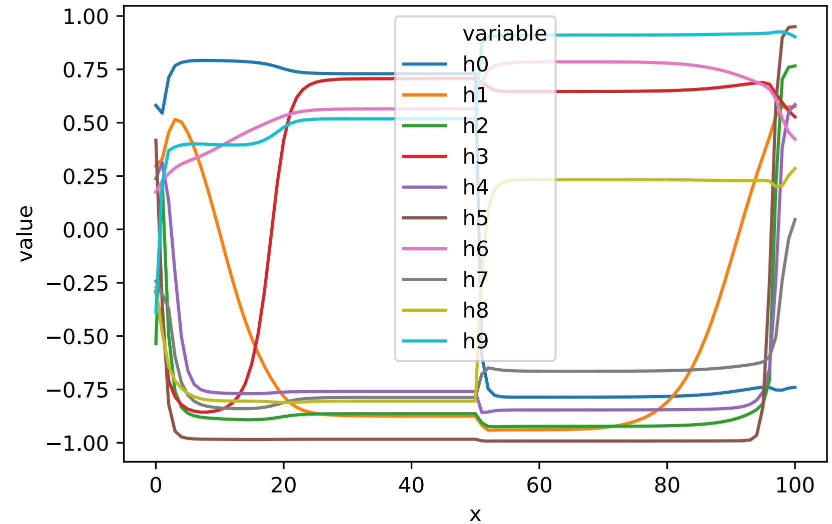


Hidden & cell state contents

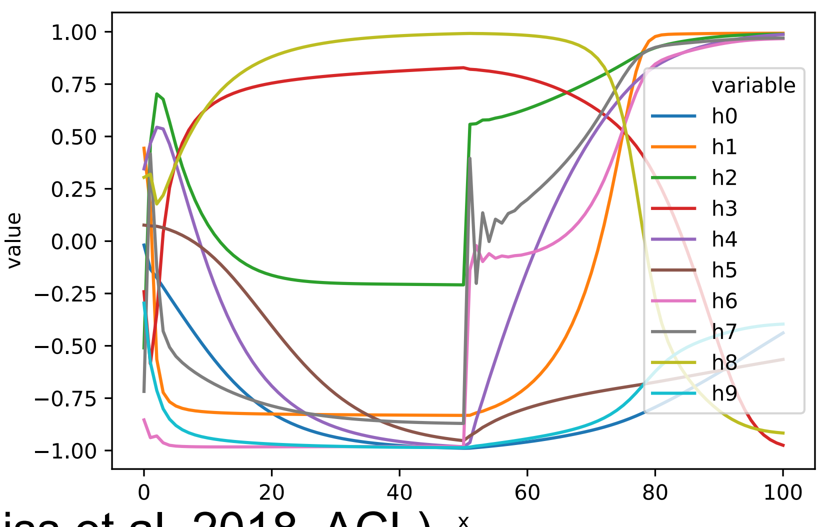
SRN hidden state



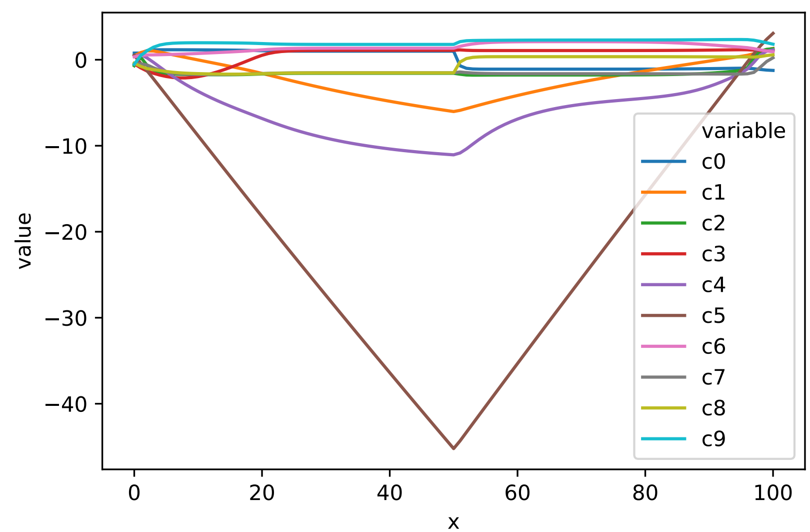
LSTM hidden state



GRU hidden state



LSTM cell state



(Weiss et al. 2018, ACL)

Summary

- Mechanisms for neural networks at the sentence level:
  - Learned word embeddings
  - Recurrent state representation
- Different units used for recurrent state representation:
  - Simple recurrent network (SRN)
  - Gated recurrent unit (GRU)
  - Long short-term memory (LSTM)
- For classic counting language, LSTM works the best